ABSTRACT

The transfer function synthesis method is one of the most powerful methods in analyzing the responses of complex built-up structures under high modal density. Its superiority mostly comes from the ability to incorporate experimental FRFs into the formulation. In this paper, a general procedure for the design sensitivity analysis of vibro-acoustic problems has been presented in frame of the transfer function synthesis method. For an acoustic response function, the proposed method gives a parametric design sensitivity expression in terms of the partial derivatives of the connection element properties and the transfer functions of the substructures. As a realistic problem, an interior noise problem in a passenger car is analyzed. The proposed noise sensitivity formulation calculates the interior noise variations with respect to the changes of the dynamic characteristics of the engine mounts and the bushes. To obtain the FRFs, a finite element model is built for the engine mount structures, and experimental FRFs are used for the trimmed body including cabin cavity. The comparison of interior noise sensitivities obtained by the proposed method and the finite difference method shows that the proposed method is efficient and accurate.

INTRODUCTION

To reach an optimum design of dynamic problems, there are many design tools. Among them, the sensitivity analysis combined with mathematical programming technique is a practical tool for design engineers when large and complicated systems are considered. The design sensitivity analysis is to study on the rate of changes in system characteristics with respect to the design parameter variation. A procedure of the design sensitivity analysis for a structural system is well summarized in Ref [1]. In dynamic problems, the design sensitivity analysis is focused on the changes of the eigenfrequencies and eigenvectors [2, 3]. They obtained the sensitivity information of a system response from the derivatives of the eigenfrequencies and eigenvectors using the modal superposition. To calculate the response sensitivity directly, researchers developed the design sensitivity formula of the frequency response functions (FRF). Akiyama et al. [4] introduced a structural modification concept in the transfer function synthesis method. Lin and Lim [5] developed a design sensitivity formula of the frequency response function from the experiments.

Dynamic substructuring technique is a method that predicts the dynamic behavior of a structure based on the dynamic behavior of the composing substructures. A method of calculating the frequency response functions of a structure composed of several substructures using the FRFs of the substructure is called as the transfer function synthesis method (TFSM) or the FRF-based substructuring method [6, 7]. The transfer function synthesis method is one of the most powerful methods in analyzing a response of complex built-up structures under high modal density. Its superiority mostly comes from the ability to incorporate experimental FRFs into the formulation. However, it cannot give the systematic guides for the structural modification or optimal design although it can predict the response of the total structure from the FRFs of the substructures. A design sensitivity analysis for the dynamic problem gives much valuable information that the designer frequently could not predict by intuition or experience. However, only a little attention is paid for the design sensitivity analysis in frame of the substructuring method. Heo et al. [8] presented a substructural sensitivity synthesis method by using component modal sensitivities. Santos et al. [9] also derived the joint stiffness sensitivity formula of a component mode synthesis model. Recently, Lallemand et al. [10] proposed a semi-analytical sensitivity analysis method in a component mode synthesis frame. As a direct approach, Jee [11] derived a response function sensitivity by differentiating the frequency response
function formed by the transfer function synthesis method with respect to the substructure receptance function. Chang et al. [12] extended Jee’s method and applied it to the structural dynamic modification.

In this paper, a general procedure for the design sensitivity analysis of the vibro-acoustic problems is presented using the transfer function synthesis formulation. By introducing the direct differentiation approach for the reaction forces on the interface elements, we derive a parametric noise sensitivity formula, in which algebraic linear equations should be solved. However, the proposed method is very powerful since the system matrix that should be inverted for the sensitivity analysis is the same with the matrix used for system response by the transfer function synthesis method. In the formula, the additional term to be calculated for the sensitivity analysis is only a vector that consists of partial derivatives of substructure FRFs and connection element properties. To verify the efficiency of the proposed method, we apply it to a realistic problem related to the engine mount system of a passenger car. The sensitivities of a response with respect to the stiffness or damping coefficients of the connecting elements are calculated and compared with those from the conventional finite difference approach.

NOISE SENSITIVITY ANALYSIS USING THE TFSM

To analyze the structure-borne noise in a passenger car, the transfer function synthesis method has advantages in viewpoint of accuracy due to its ability to combine the experimental FRFs with the numerical ones. Here the transfer function synthesis method is summarized briefly.

Consider a vibro-acoustic system with two substructures connected by the mechanical elements such as springs and dampers in Figure 1. When an external force $F$ excites the substructure A, the response at point $r$ on the substructure B will be derived. The $k_i$ and $C_i$ represent the stiffness of spring and the damping coefficient of damper in the $i$th connection element, respectively. The number of connection points is $n$ along the interface boundary. Hereafter we adopt the summation convention in the indicial notation. The displacements of the connection points on the substructure A, $x^i$, can be written as follows.

$$x_i^i = H^i R^i + H^i F, \quad i = 1, \ldots, n \quad (1)$$

where $R^i$ is the reaction force of the $i$th connection point and $H^i$ the frequency response of the $i$th point when a unit force excites the $i$th point. For the substructure B, we can write the displacements of the connection points as

$$x_i^0 = -H^i R^i, \quad i = 1, \ldots, n \quad (2)$$

where the directions of the reaction forces are reversed to be consistent with equation (1). Neglecting the airborne noise and excitations by the wind, the acoustic response at the point $r$ in the substructure B, $p^r$, is

$$p^r = -H^i R^i \quad (3)$$

where $H^i$ is the noise transfer function of the response point when a unit force is exerted at the $i$th connection point. The substructures are connected by a number of elastic springs and viscous dampers. The compatibility conditions between the substructures along the interface give us the following relations:

$$H^i R^i = x_i^0 - x_i^f, \quad i = 1, \ldots, n \quad (4)$$

where

$$H^i = \frac{1}{k_i + j \omega C_i}, \quad \text{if } i = j$$

$$= 0, \quad \text{if } i \neq j$$

and $\omega$ is the angular velocity. The reaction forces are derived by the substitution of the equations (1) and (2) into (4):

$$R^i = -D^i H^i F^i, \quad i = 1, \ldots, n \quad (5)$$

where

$$D^i = H^i + H^i + H^i \quad (6)$$

Finally the acoustic response in the substructure B can be obtained by plugging equation (5) into equation (3) as follows.

$$p^r = H^i D^i H^i F \quad (7)$$

Using equation (7), we can predict the interior noise level in a passenger car from the frequency response functions of the subsystems and the interface conditions.

To obtain the variation of the acoustic response function due to a design change, we need the gradient information with respect to the design variable, which comes from the design sensitivity analysis. The first step of design sensitivity analysis is to express the variation of response as an explicit equation to the design change. Differentiating equation (3) with respect to the design variable gives

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Figure 1. A vibro-acoustic system
\[
\frac{dp_x}{db} = - \frac{\partial H_x^*}{\partial b} \cdot R - H_x^* \cdot \frac{\partial R}{\partial b} \quad (8)
\]

where \(b\) is the design variable such as the stiffness or the damping coefficient of the connection element. In equation (8), the first term on the right hand side can be computed by the conventional formulations \([13-17]\). However, the second term, \(\partial R / \partial b\), is not explicit to the design change because the reaction forces are determined by the dynamic characteristics of assembled system. To obtain an explicit expression of \(\partial R / \partial b\), we start from the equation (5). Multiplication of \(D_x\) on each side of equation (5) results in,

\[
D_x R = -H_x^* F
\]

By differentiating the above equation with respect to the design variable, equation (9) can be written as

\[
D_x \frac{\partial R}{\partial b} = - \frac{\partial D_x}{\partial b} \cdot R_j - \frac{\partial H_x^*}{\partial b} \cdot F - H_x^* \cdot \frac{\partial F}{\partial b} \quad (10)
\]

Since all terms in the right hand side of equation (10) are the known functions, we can solve linear algebra equations to obtain the variation of the reaction forces with respect to the design variable. Note that there is no additional cost to get the inverse system matrix, \(D_x\), since we already got it during the calculation of responses in equation (5). Since the external force is independent upon the design variable generally, we can rewrite the noise sensitivity formula by replacing \(\partial R / \partial b\) in equation (8) with equation (10),

\[
\frac{dp_x}{db} = - \frac{\partial H_x^*}{\partial b} \cdot R + H_x^* D_x^{-1} \left\{ \frac{\partial D_x}{\partial b} \cdot R_j + \frac{\partial H_x^*}{\partial b} \cdot F \right\} \quad (11)
\]

Since the design sensitivity expression (11) is explicit to the design variables, it can be calculated by the simple numerical manipulations. The additional calculations to get the sensitivity information are those of only three terms, that is, \(\partial D_x / \partial b\), \(\partial H_x^* / \partial b\), \(\partial H_x^* / \partial b\) and several matrix multiplications. This feature makes the design sensitivity formulation efficient. Moreover, the change of a design variable in one subsystem will not affect the dynamic characteristic of the others in the substructuring approach, which makes many terms zero in the noise sensitivity formula.

Frequently, the properties of the connection element, i.e. the stiffnesses and damping coefficients of the engine mounts and bushes have a significant influence on the noise and vibration problems of a passenger car. In this case, \(\partial D_x / \partial b\) equals to \(\partial H_x^* / \partial b\), which can be evaluated by a differentiation of the analytic expression of \(H_x^*\), equation (4). Other derivative terms in equation (11) are zero. A typical example of this case is an optimization problem of automotive engine mount system and will be treated in the numerical example.

Note that it is not difficult to expand the proposed formulation to general complex systems. We can get the sensitivity information for a system with multiple substructures, excitations or responses, although we considered a relatively simple structure in the derivation.

**ENGINE MOUNT PROBLEM**

The interior noise level in a passenger car can be predicted by the transfer function synthesis method. To analyze the structure-borne noise in the cavity of a car due to engine excitation, the car is divided into two substructures. Substructure A contains the power train and the sub-frame, and substructure B is a trimmed body structure including the cabin cavity. The excitation is the bouncing force and rolling moment at the engine due to firing. The response concerned is the sound pressure level at the passenger’s ear position. Two engine mounts and five rubber elements connect two substructures as shown in Figure 2. The sensitivities of the interior noise level with respect to the properties of engine mounts and bushes are very important to reduce the interior noise level.

The transfer function synthesis method can use the experimental or the analytical data depending on the characteristics of substructures. To obtain the FRFs of the substructure A, we use a finite element analysis. Through the frequency response analysis in MSC/NASTRAN, the FRFs are calculated. For the substructure B, the noise transfer functions from the connection point to the passenger’s ear position are
measured experimentally by the impact hammer test. In general, rubber elements show nonlinear frequency characteristics, which should be considered in the interior noise analysis. The hydraulic engine mount has highly frequency dependent responses especially. This feature leads the engine mount problem to a nonlinear modal analysis problem, which is one of barriers of the modal superposition approach. In this research, we use the stiffness and damping coefficients of the connection elements experimentally measured by the elastomer tester. To identify the engine firing forces, combustion pressures with respect to angle of crankshaft in a cylinder are measured and converted to external force acting on the engine block. Figure 3 shows the sound pressure level at the ear position calculated by the transfer function synthesis method. In the calculation the reaction forces at the muffler hanger points and shock absorber points are neglected since their contributions to the interior noise are negligible.

The interior sound pressure level shows a high peak around 1800 rpm, which is identified as a structure-borne noise due to the engine excitation. To investigate the influences of the engine mounts and sub-frame bushes on the interior sound pressure level systematically, the proposed noise sensitivity analysis method is applied. A target response is the interior sound pressure at the passenger’s ear position. The design variables are the stiffness and the damping coefficient of every engine mount and bush between the substructure A and B. There are six design variables at each connection point since three orthogonal directions must be counted separately. The number of total design variables is forty-two since there are seven points between the two substructures as shown in Figure 2.

Figure 4. Noise sensitivity results w.r.t. the stiffnesses in X-direction

Figure 5. Noise sensitivity results w.r.t. the stiffness in Y-direction

Figure 6. Noise sensitivity results w.r.t. the stiffnesses in Z-direction.
Figures 4 to 6 show the sensitivities of the interior noise level with respect to the stiffnesses of each connection element. Note here that the relative phase of the noise sensitivity to the interior noise as well as the magnitude of the noise sensitivity is important because the overall change of the acoustic response is the vector sum of the sound pressure level and the noise sensitivity multiplied by the design change. As seen in Figure 4 to 6, the interior sound pressure level is very sensitive to the stiffness changes at the connection point five in all directions. The next sensitive design variables are the stiffness of the connection points nine and ten. The calculated design sensitivities at each point are compared with those by the finite difference method in Figure 7. For the finite difference method, each design variable is perturbed by 0.01% of the present value. Although the design sensitivities are complex values, only the magnitudes of them are compared for brevity. The comparison shows excellent agreements between two methods and the maximum errors are less than 0.1%. In Figure 7(b) and (c) the errors of the connection point five are relatively large compared with the others over some frequency range. This is due to the fact that the amount of perturbation is constant w.r.t. the current value although the relative magnitude of the noise sensitivity is very large compared with the others over that frequency range as shown in Figure 5 and 6. Actually we can see that the smaller the perturbation size in the finite difference method, the smaller sensitivity errors of that points.
the design modification.

explains the effectiveness of the sensitivity analysis for is more effective to decrease the noise level. This

nine and ten. However, the stiffness change at point five

not decrease so much by reducing the stiffness of point

applied respectively. As expected, the noise level does

close to the response vector, which means the design

magnitude and the phase of the sensitivity vector as

expected from the formulation in equation (4). In Figure

nine and ten are modified by +15%, -15% and – 15% of

levels are calculated when the stiffness of elements five,

effectively. To verify these results, the interior noise

change at this element can modify the response

sensitivities with respect to the stiffness of elements ni ne and ten are

nearly perpendicular to the response vector. This means

that the stiffness changes of these elements do not

change the response so much. On the contrary, the

sensitivities with respect to the stiffness of elements five

in Z-direction has similar magnitude but the direction is

similar to those of the stiffnesses except the absolute

magnitude and the phase of the sensitivity vector as

expected from the formulation in equation (4). In Figure

9, the sensitivities from the proposed method and the

finite difference method are compared. The sensitivities

calculated by the proposed method show good accuracy

and the maximum error of the damping coefficient

sensitivities is less than 0.01 %. This shows that the

proposed method can calculate the sensitivities in a

simple and easy way even for a complicated real

problem.

In order to identify the design variables that have large

influence on the interior noise for the peak around 1800

rpm, the sensitivity vectors at 1800 rpm are plotted as

shown in Figure 10. In the sensitivity polar plot, the
direction as well as the magnitude of the sensitivity is

very important since the expected amount of variation
due to the design change is proportional to the projected

magnitude of the sensitivity vector to the direction of the

response vector. As shown in Figure 10, the sensitivities

with respect to the stiffness of elements nine and ten are

large in magnitude, but the directions of the vectors are

nearly perpendicular to the response vector. This means

that the stiffness changes of these elements do not

change the response so much. On the contrary, the

sensitivities with respect to the stiffness of elements five

in Z-direction has similar magnitude but the direction is

close to the response vector, which means the design

change at this element can modify the response

effectively. To verify these results, the interior noise

levels are calculated when the stiffness of elements five,
nine and ten are modified by +15%, -15% and –15% of

the present value in all directions, respectively. Figure 3

shows the interior noise level when the modifications are

applied respectively. As expected, the noise level does

not decrease so much by reducing the stiffness of point

nine and ten. However, the stiffness change at point five

is more effective to decrease the noise level. This

explains the effectiveness of the sensitivity analysis for

the design modification.

Figure 8 shows the sensitivity results with respect to the
damping coefficients of the connection elements. The
sensitivity results of the damping coefficients look very
similar to those of the stiffnesses except the absolute
magnitude and the phase of the sensitivity vector as
expected from the formulation in equation (4). In Figure
9, the sensitivities from the proposed method and the
finite difference method are compared. The sensitivities
calculated by the proposed method show good accuracy
and the maximum error of the damping coefficient
sensitivities is less than 0.01 %. This shows that the
proposed method can calculate the sensitivities in a
simple and easy way even for a complicated real
problem.

CONCLUSION

The sensitivity analysis is very useful in the trouble
shooting or design problem of the vibro-acoustic
phenomena. For large and complex structures such as
full vehicle, the FRF-based substructuring approach is
known as one of the most powerful tools in analyzing the
system response. In this paper, a general procedure for
the design sensitivity analysis of the vibro-acoustic
problems in frame of the FRF-based substructuring
formulation has been presented. The present methods
can give us a systematic design guide when we analyze
a structure-borne noise by the transfer function synthesis
method. For an acoustic response function, the
proposed method gives a parametric design sensitivity
formula in terms of the partial derivatives of the
connection element properties and the transfer matrixes
of the subsystems. The design sensitivities with respect
to the engine mounts and bushes in an engine mount
system are calculated by the present method and
compared with those by the finite difference method. The
comparison shows that the proposed method can
calculate the design sensitivities accurately even for a
complicated real problem.

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