DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION OF AN ENGINE MOUNT SYSTEM USING THE FRF-BASED SUBSTRUCTURING

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Abstract

A general procedure for the design sensitivity analysis of acoustic response function has been presented using the FRF-based substructuring formulation. The design sensitivity formulation is derived based on the governing equations of the FRF-based substructuring method. The derived noise sensitivity formula is combined with a nonlinear programming module to obtain the optimal design for an engine mount system of passenger car. The object function is the area of the interior noise graph over a concerned rpm range. The interior noise variations with respect to the dynamic characteristics of the engine mounts and bushings have been calculated using the proposed sensitivity formulation and transferred to the nonlinear optimization software. To obtain the FRFs, we use a finite element analysis for the engine mount structures and experimental techniques for the trimmed body including cabin cavity. The optimization based on the sensitivity analysis gives the optimal stiffnesses of the engine mount and bushings. The optimized interior noise in the passenger car shows that the proposed method is efficient and accurate.

INTRODUCTION

Researchers have proposed a lot of methods to analyze the vibro-acoustic coupled problem. The finite element analysis is one of the most popular tools [1] since it can solve many types of engineering problems systematically. However, the complexity of the vibro-acoustic problem such as the interior noise in a car cabin makes the finite element approach limited due to its high computational cost and low accuracy. As another approach, dynamic substructuring methods, which can combine the experimental data with the numerical ones, are suggested. In addition, the loads for the computation are less than others in the design iterations [1~3]. There are two categories in the dynamics substructuring method. One is the modal coordinate approach. The other is the transfer function synthesis method. The modal method has advantages in computational cost due to small degree of freedom of the modal coordinate, and the transfer function method is proper to high modal density system because the measured frequency response function contains exact system characteristics even in case of the
high modal density system.

A nonlinear programming algorithm coupled with the sensitivity information can give an optimal design of the engine mount system systematically. The gradient-based optimization algorithms need the design sensitivity information for the cost function and constraints, which is the rate of change with respect to the design variable [4]. Various methods and techniques have been presented to calculate the design sensitivity efficiently in frame of the dynamic substructuring method such as modal approaches [5–7] and transfer function approaches [8, 9]. In the recent study [10], the authors have presented a general procedure for the design sensitivity using the FRF-based substructuring formulation and proven that the method is not only accurate but also efficient.

In this study, optimal stiffness of the engine mounts and bushings are obtained using the FRF-based substructuring method. To obtain the design sensitivity of interior sound pressure level, a parametric sensitivity formula is derived for the structural-acoustic coupled system. Then an optimization problem is defined for the engine mount system of the real passenger car, and the optimal solution is obtained by plugging the derived sensitivity formula into a nonlinear optimization software.

DESIGN SENSITIVITY ANALYSIS USING THE FBSTM

Consider a linear structural-acoustic system with two substructures connected by the mechanical elements such as springs and dampers in Figure 1. The substructure B contains a cavity surrounded by flexible wall. When external forces $F^A$ and $F^B$ excite the substructure A and the substructure B, respectively, the sound pressure level at point $r$ in the cavity of the substructure B is the target response to be minimized. The $k_i$ and $C_i$ represent the stiffness of spring and the damping coefficient of damper in the $i$-th connection element, respectively. The number of connection points is $n$ along the interface boundary. Hereafter we adopt the summation convention in the indicial notation. The displacements of the connection points on the substructure A, $x_i^A$, can be written as follows.

$$x_i^A = H_{ij}^A R_j + H_{ij}^A F^A, \quad i = 1, \cdots, n$$

(1)

where $R_j$ is the reaction force of the $j$-th connection point and $H_{ij}^A$ the receptance frequency response of the $i$-th point when a unit force excites the $j$-th point. For the substructure B, we can write the displacements of the connection points as

$$x_i^B = -H_{ij}^B R_j + H_{ij}^B F^B, \quad i = 1, \cdots, n$$

(2)

where the directions of the reaction forces are reversed to be consistent with equation (1). The sound pressure level at the point $r$ in the cavity of the substructure B is the summation of contributions due to the external forces acting on the substructure B and written as

$$p_r^B = -H_{rj}^B R_j + H_{rj}^B F^B$$

(3)

The substructures are connected by a number of elastic springs and viscous dampers. The compatibility conditions are satisfied at the interface boundary.
between the substructures along the interface give us the following relations:

\[ H_i^j R_j = x_i^b - x_i^A, \quad i = 1, \ldots, n \]  \hspace{1cm} (4)

where \( H_i^j = \frac{1}{(k_i + \sqrt{-1} \omega C_i)}, \quad \text{if} \quad i = j \)

\[ = 0, \quad \text{if} \quad i \neq j \]

and \( \omega \) is the angular velocity. The reaction forces are derived by the substitution of the equations (1) and (2) into (4):

\[ R_i = D_q^i (H_q^B F^B - H_q^A F) \quad i = 1, \ldots, n \]  \hspace{1cm} (5)

where \( D_q^i = H_q^i + H_q^A + H_q^B \)  \hspace{1cm} (6)

Finally the sound pressure level in the cavity of the substructure B can be obtained by plugging equation (5) into equation (3) as follows:

\[ p_r^B = H_n D_q^i (H_q^A F^A - H_q^B F^B) + H_q^B F^B \]  \hspace{1cm} (7)

Using equation (7), we can predict the sound pressure response of a system from the frequency response functions of the subsystems and the interface properties. This is a brief summary of the FRF-based substructuring method for the structural-acoustic system.

To obtain the variation of the response function due to a design change, we need the gradient information with respect to the design variable, which comes from the design sensitivity analysis. Differentiating equation (3) with respect to the design variable gives

\[ \frac{dp_f^B}{db} = -\frac{\partial H_n^B}{\partial b} \cdot R_i - \frac{\partial H_q^B}{\partial b} \cdot \frac{\partial R_i}{\partial b} + \frac{\partial H_q^B}{\partial b} \cdot F^B + \frac{\partial H_q^B}{\partial b} \cdot \frac{\partial F^B}{\partial b} \]  \hspace{1cm} (8)

where \( b \) is the design variable such as the stiffness or the damping coefficient of the connection element. In equation (8), all terms except the second one on the right hand side can be computed by the conventional formulations [11]. However, the second term, \( \frac{\partial R_i}{\partial b} \), is not explicit to the design change because the reaction forces are determined by the dynamic characteristics of assembled system. To obtain an explicit expression of \( \frac{\partial R_i}{\partial b} \), we start from equation of the reaction force, equation (5). Multiplication of \( D_q^i \) on each side of equation (5) results in,

\[ D_q^i R_j = -H_q^A F^A + H_q^B F^B \]  \hspace{1cm} (9)

By differentiating the above equation with respect to the design variable, equation (9) can be written as

\[ D_q \cdot \frac{\partial R_j}{\partial b} = -\frac{\partial D_q^i}{\partial b} \cdot R_j - \frac{\partial H_q^A}{\partial b} \cdot F^A + \frac{\partial H_q^A}{\partial b} \cdot \frac{\partial F^A}{\partial b} + \frac{\partial H_q^B}{\partial b} \cdot F^B + \frac{\partial H_q^B}{\partial b} \cdot \frac{\partial F^B}{\partial b} \]  \hspace{1cm} (10)

Since all terms in the right hand side of equation (10) are the known functions, we can solve linear algebra equations to obtain the variation of the reaction forces with
respect to the design variable. Note that there is no additional cost to get the inverse system matrix, $D_y^B$, since we already got it during the calculation of responses in equation (5). Since the external forces are independent upon the design variables generally, we can rewrite the design sensitivity formula by replacing the equation (10) into equation (8),

$$\frac{dp^B}{db} = -\frac{\partial H^B}{\partial b} R_i + H^B D_y^B \left\{ \frac{\partial D^B}{\partial b} R_k + \frac{\partial H^A}{\partial b} F_A - \frac{\partial H^B}{\partial b} F^B \right\} + \frac{\partial H^B}{\partial b} F^B \quad (11)$$

Since the design sensitivity expression (11) is explicit to the design variables, it can be calculated by the simple numerical manipulations. The additional calculations to get the sensitivity information are partial derivatives of the receptance transfer functions and several matrix multiplications. Note that many of the partial derivatives of the receptance function will vanish in the design sensitivity calculation because the change of a design variable in one subsystem will not affect the dynamic characteristic of the others in the substructuring approach. This feature makes the design sensitivity formulation very efficient.

**OPTIMIZATION OF ENGINE MOUNT SYSTEM**

The design sensitivity formulation presented in the previous section is applied in order to optimize the engine mount system of passenger car systematically.

To analyze the interior noise level using the FRF-based substructuring method, a passenger car is partitioned into two substructures and connection elements as shown in Figure 2. The substructure A contains the engine and transmission assembly and subframes. The substructure B consists of the trimmed body structure including the cabin cavity. The characteristics of the connection elements connecting two substructures are the design space of the present optimization problem. Two engine mounts and five bushings connect two substructures as shown in Figure 3. In the problem, external force on the substructure B such as road-induced forces is not considered because the problem is focused on the structure-borne noise due to the powertrain vibration.

Figure 4 shows the interior noise of the initial design calculated by the FRF-based substructuring method. To analyze the interior noise using equation (7), FRFs from an FE model of the substructure A are combined with the measured FRFs of the substructure B. The excitation force is obtained from the measured pressure of the
Figure 4. Sound pressure level of cabin cavity at the initial design.

noise level over a fixed rpm range. An objective function selected here is the area of the interior noise graph in the decibel scale. To emphasize on high-level peaks in the objective function, only the area formed by the noise graph and a target level is considered as shown in Figure 4. Therefore the objective function is written as

\[
 f(b) = \int \omega h(\omega) \cdot \langle p^* \rangle \cdot p^* d\omega \\
 \text{where } p^* = 20 \cdot \log_{10} \frac{\sqrt{\text{Re}(p(\omega))^2 + (\text{Imag}(p(\omega)))^2}}{P_{\text{ref}}} - P_{\text{target}}
\]  

(12)

Here, \( b \) is the design vector and \( f \) is the objective function. \( \langle \cdot \rangle \) is a step function, that is one for the positive argument, zero for the negative one. \( P_{\text{target}} \) is the target value of the interior noise level in the decibel scale, \( \omega \) the frequency and \( h \) a user-defined weighting function. \( \text{Re} \) and \( \text{Imag} \) mean the real and imaginary values, respectively.

In the design stage of the engine mount system, designer can select freely the stiffness of the engine mounts and bushings whereas the damping coefficients frequently can not be prescribed as design specification since the damping coefficient of the mounts and bushings are only dependent on the material itself. Therefore we select only the stiffnesses at the connection elements as design variables. Since the connection elements can have different stiffnesses in three orthogonal directions, twenty-one design variables are defined for the engine mount system. In addition the bushings 4, 5, 6 and 7 are assumed to be axisymmetric properties to consider the productivity, so that the stiffnesses in x-direction and y-direction keep the same value during the design iterations.

In summary, the design objective of the engine mount optimization problem is to determine the stiffnesses of the engine mounts and bushings such that the objective function is minimized under the constraints of the axisymmetric and move limits of the design variables. Therefore a design optimization problem is defined as follows.

\[
 \text{Find } b \text{ such that} \\
 \text{minimize } f(b) \\
 \text{subjected to } b(I(k)) = b(J(k)), \ k = 1, \ldots, n \\
 b_L \leq b(i) \leq b_U, \quad i = 1, \ldots, m
\]

(13)

where \( b_L \) and \( b_U \) are the lower and upper bounds of the design variables,
Figure 5. Sensitivity of the interior noise w.r.t. the stiffness of the connection elements in z-direction; (a) Real part, (b) Imaginary part.

respectively. Here \( I = \{10, 13, 16, 19\} \), \( J = \{11, 14, 17, 20\} \), and \( m \) and \( n \) are 21 and 4, respectively. The gradient of the objective function is obtained from the equation (12) analytically as

\[
\frac{\partial f}{\partial b} = C^* \int_{\omega_1}^{\omega_2} h(\omega) \cdot (p^*) \left[ \frac{\text{Re}(p(\omega)) \cdot \text{Re}(\frac{\partial p}{\partial b}) + \text{Imag}(p(\omega)) \cdot \text{Imag}(\frac{\partial p}{\partial b})}{\|p\|^2} \right] d\omega
\]

(14)

where \( C^* = 20 \cdot \log_{10} \epsilon \) and \( \| \| \) means the magnitude of vector. The design sensitivities of the interior noise with respect to the design variables, \( \frac{\partial p}{\partial b} \) in the above equation are calculated by the presented design sensitivity formula, equation (11). The sensitivity information of the other constraints is obtained by direct differentiation of the analytic equation.

To get the optimum design of the engine mount system in equation (13), the proposed design sensitivity analysis method is plugged into commercial optimization software, MATLAB. The current stiffness values normalize the design variables and the upper and lower bounds are 1.3 and 0.7, respectively. Figure 5 shows the sensitivities of the interior noise with respect to the stiffnesses of the connection elements in z-direction. Note that both the interior noise and the design sensitivities are the vector quantity composes of the real and the imaginary components. In addition, the noise sensitivity with respect to the design variable is frequency-dependent as shown in Figure 5, and we can see, for instances, that the sensitivity of the element three has large influence over the concerned rpm range but the bushings six and seven are significant over a limited rpm range.

The optimum design to minimize the interior noise level is obtained using the MATLAB software when the concerned rpm range is from 1000 to 4000 rpm. Two different weight functions are introduced. Those for the first case (CASE I) are one over the entire concerned rpm range. Those for the second case (CASE II) are also one but two for the range from 1100 to 2200 rpm to concentrate on reduction of the greatest peak around 1800 rpm. To verify the

Figure 6. Sound pressure levels of the cabin cavity at the optimum designs
Table 1 Design sensitivity result of the objective function at the initial design ($f^* = 385.80$)

<table>
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<tr>
<th>Design Var. No</th>
<th>Present Method $f'$</th>
<th>FDM $\delta f = \Delta f / \Delta b$</th>
<th>Ratio [%] $(f' / \delta f) \times 100$</th>
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sensitivity formulation in section 2, the sensitivity results of the objective function for the first case are compared with those from the forward finite difference method. For the finite difference method, amount of perturbation is 0.1% of the current design value. Two results in Table 1 show very good agreements, which proves that the presented design sensitivity formula is correct and accurate. Figure 6 shows the interior noise level of the optimum design. The interior noise levels minimized with different weight functions are almost identical except around 3500 rpm although some design variables have converged to the different values. Note also that the highest peak around 1800 rpm at the optimum design has lowered by more than 3 dB compared to the initial design. The cost function for the first case has reduced from 385.8 to 300.2 in 29 iterations and that for the second case from 648.3 to 512.2 in 34 iterations. The objective functions decrease by the amount of 22.2% and 21.00%, respectively.

CONCLUSION

A general procedure for the design sensitivity analysis of structural-acoustic
problems has been presented in frame of the FRF-based substructuring formulation. Since the FRF-based substructuring method can model the whole system from FRFs of the substructures, it is very efficient tool to analyze the complex system like cars. For an acoustic response function, the proposed noise sensitivity analysis method gives a parametric design sensitivity expression in terms of the partial derivatives of the connection element properties and the transfer functions of the components. The optimal design for an engine mount system of passenger car is obtained using the derived noise sensitivity formula with a nonlinear programming software. The object function is the area of the interior noise graph over the concerned rpm range. The interior noise variations with respect to the stiffnesses of the engine mounts and bushings have been calculated using the proposed sensitivity formulation and transferred to the nonlinear optimization software to obtain the optimal stiffnesses of the engine mount and bushings. The optimized interior noise in the passenger car shows that the proposed method gives accurate design sensitivity information efficiently for the structural-acoustic systems and could be used to determine the characteristics of the engine mount systems in the system level design.

In the present analysis, two engine mounts are not included as design variable since they are parts of a substructure. Extension of the present formulation to the multiple substructure system will include any mounts in the design variables. This is a challenge for future work.

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