

Shading

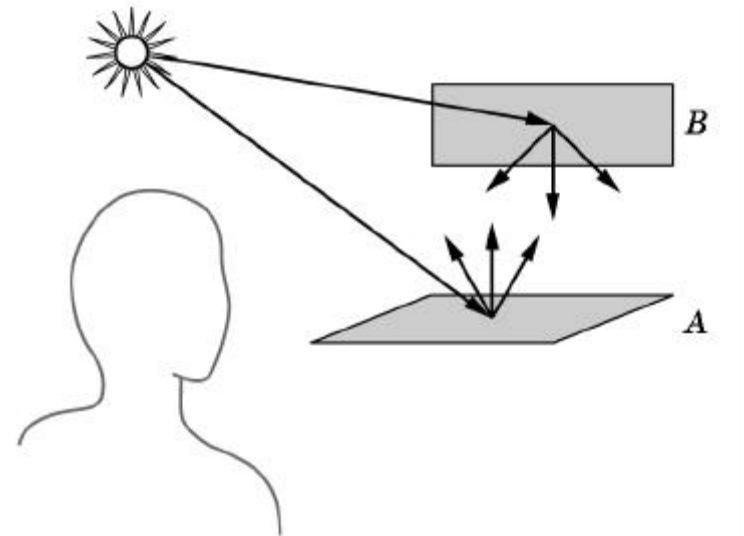
Shading

- realistic computer graphics
 - gradation of colors

-

- what is needed ?

가

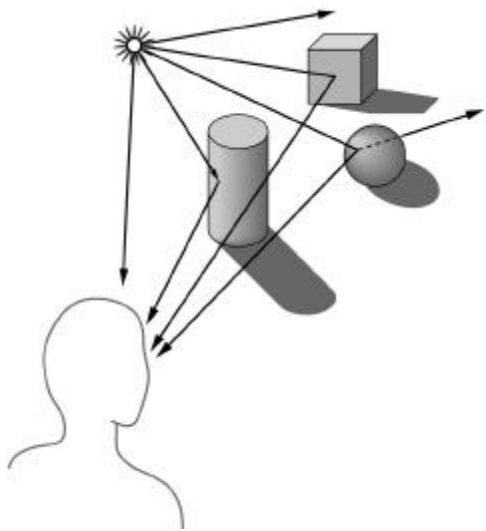


- light :
- matter (= material) :
- optics () or physics

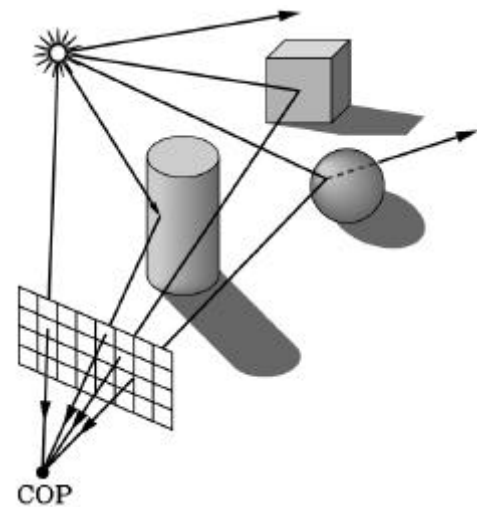
6.1 Light and Matter

Light and Matter

- **Light** representation in computer graphics
 - based on three color theory : Red, Green, Blue
- **Matter**
 - material properties determine which wavelengths are absorbed or reflected to some extent



in real world



in computer graphics

Rendering Equation

- J. T. Kajiya, “The Rendering Equation”, *SIGGRAPH* '86, pp.143–150, (1986).
 - computer graphics
 - optics approximation
- rendering equation
 - illumination model

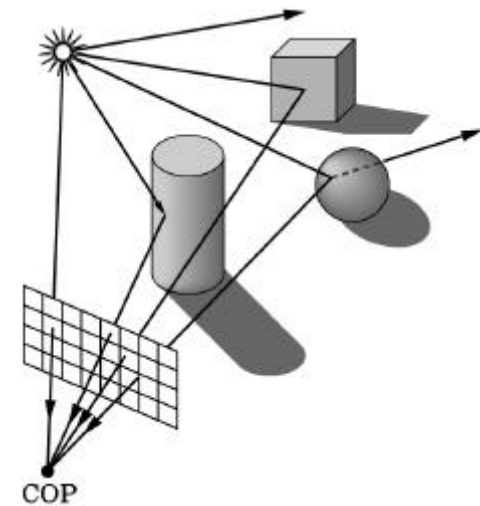
light source, matter,

rendering equation

pixel

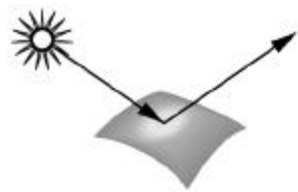
rendering

가



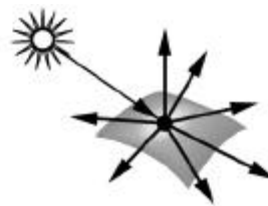
Illumination Model

- = shading model, lighting model
 - a method to calculate the intensity of light at a given point on the surface of an object
- Light-Material interactions
 - material
 - ...



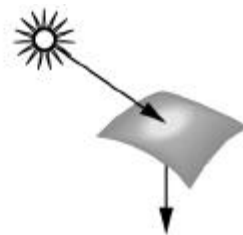
(a)

specular surface



(b)

diffuse surface



(c)

translucent surface

6.2 Light Sources

Light Sources

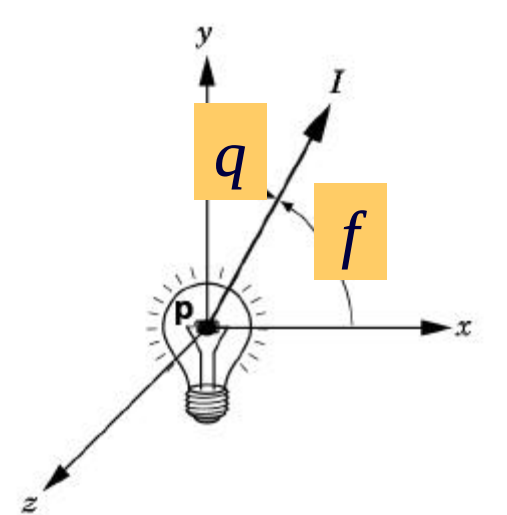
- light
 - self-emission : ()
 - reflection :
- light source (光原)

- :
- :

- illumination function

$$I(x, y, z, q, f, l)$$

- point (x, y, z)
 (q, f) 가
 l

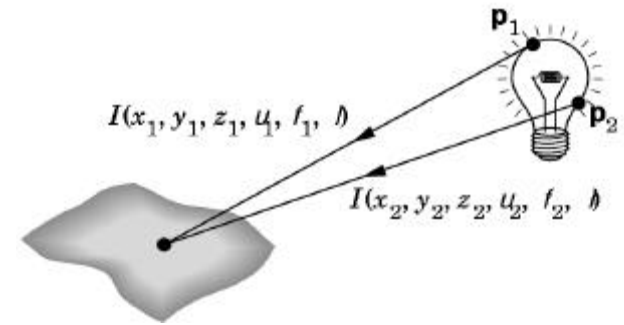


Light Sources

- additive behavior of light
 - : integral of all lights
 - color : R, G, B

$$I = \begin{bmatrix} I_r \\ I_g \\ I_b \end{bmatrix} = \begin{bmatrix} I_r(x, y, z, q, f, l) \\ I_g(x, y, z, q, f, l) \\ I_b(x, y, z, q, f, l) \end{bmatrix}$$

- four types of light sources
 - ambient lighting
 - point light source
 - spotlight
 - distant light



Ambient Light

- uniform lighting (typically in the room)

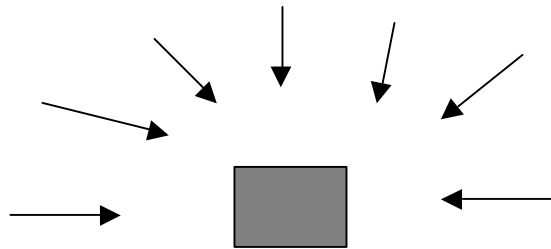
- the combination of light reflections from various surfaces

-

- : 가

- ambient light 가 가 , 가

- ambient illumination : a scalar value I_a

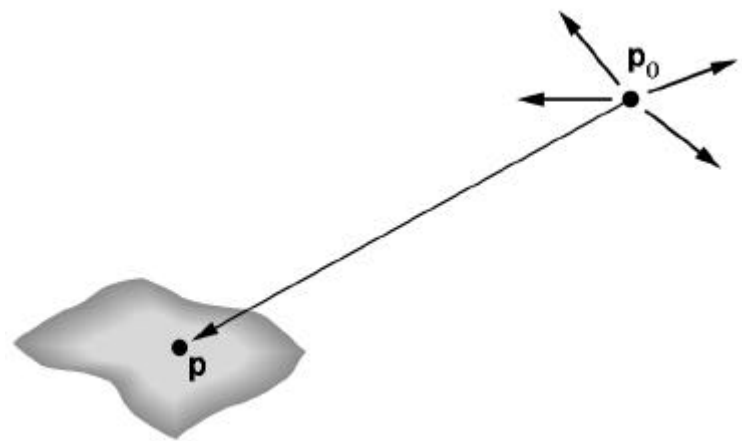


light

Point Light Sources

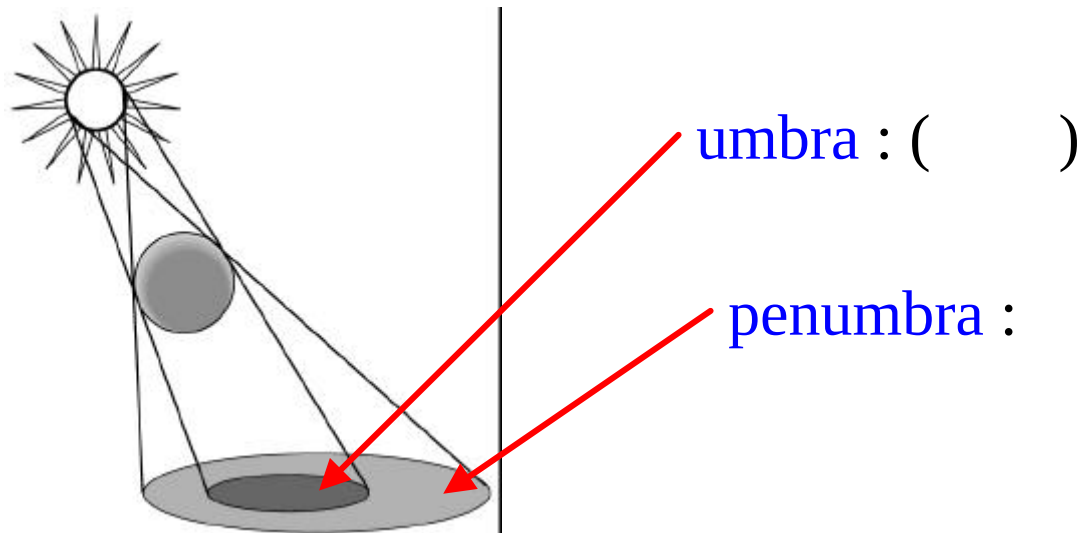
- ideal point light source
 - $\mathbf{p}_0$
 - emits light equally in all directions
- light received from the point light source
 -

$$I(\mathbf{p}, \mathbf{p}_0) = \frac{1}{|\mathbf{p} - \mathbf{p}_0|^2} I(\mathbf{p}_0)$$



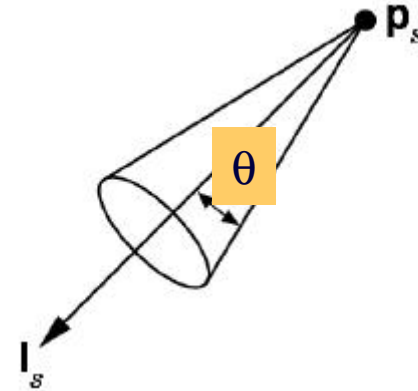
Point Light Sources

- point light source in real world
 - :
 - not point, but **area light source**
 -



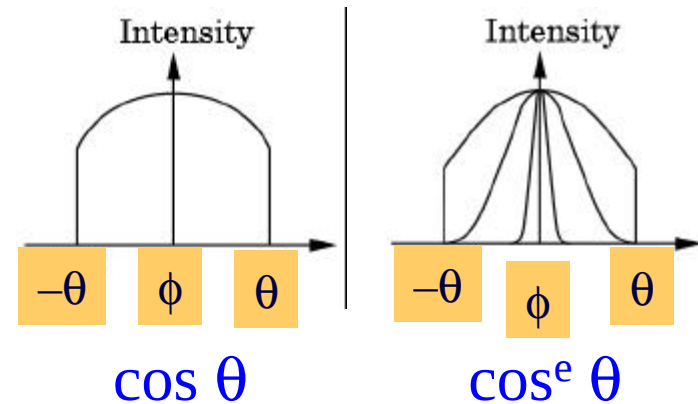
Spotlights

- cone emit
 - $\mathbf{p}_s$: apex ()
 - $\mathbf{I}_s$: light direction
 - θ : angle
 - point light source : $\theta = 180^\circ$



- more realistic case

$$f \propto \cos^e \theta$$



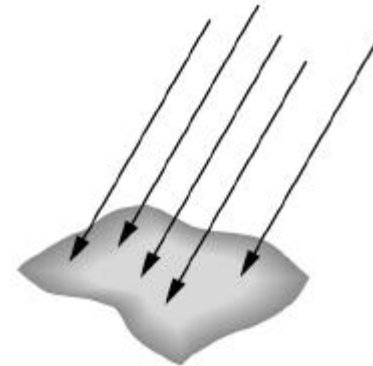
Distant Light Sources

- = parallel light source
 -
 - :
 - :
 - in homogeneous coordinate,

$$\mathbf{p}_0 = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \mathbf{p}_0 = \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

point light source

parallel light source



6.3 The Phong Reflection Model

Phong Reflection Model

- Bui-T. Phong, “Illumination for Computer Generated Pictures”, *Comm. ACM*, 18(6):311–317, (1975).

- local illumination model

–

,

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–

가 !

normal vector

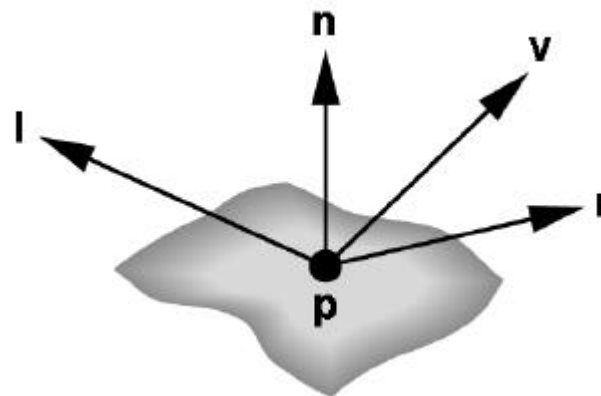
light direction

view direction

– ambient term

– diffuse term

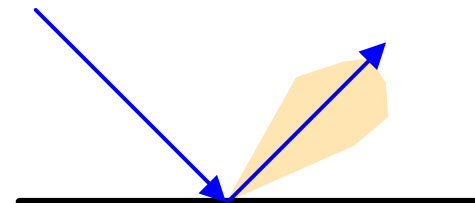
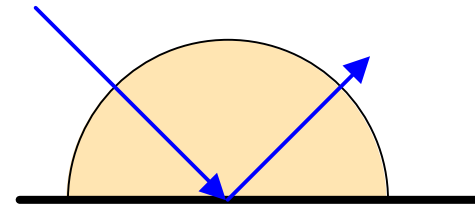
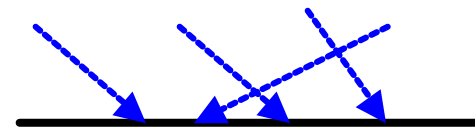
– specular term



reflection

Phong Reflection Model

- reflection on the surface
 - ambient illumination
 - indirect reflection approximate
 - diffuse illumination
 -
 - specular illumination
 -

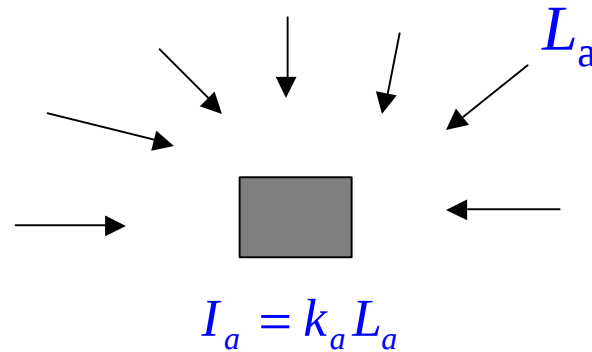


Ambient Reflection

- ambient reflection I_a
 - 가?
 - a surface that is not exposed directly to a light source still will be visible if nearby objects are illuminated
- L_a : ambient light
- k_a : ambient reflection coefficient

$$0 \leq k_a \leq 1$$

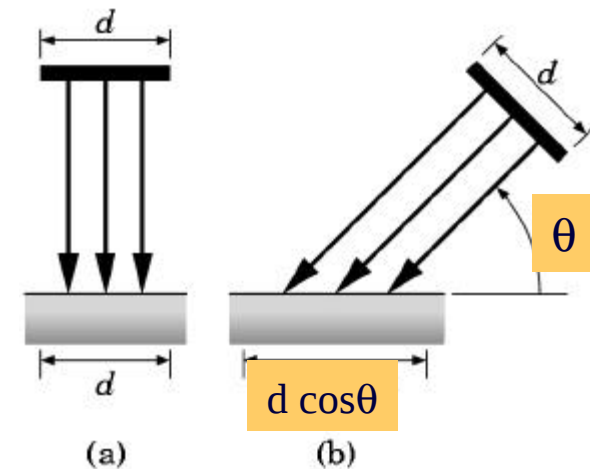
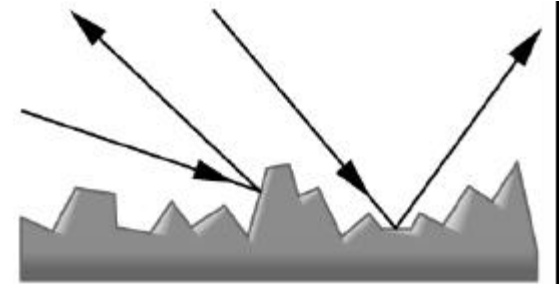
$$I_a = k_a L_a$$



Diffuse Reflection

- (亂)
- ,
- Lambertian surface :
- $I_d = k_d L_d \cos\theta = k_d (\mathbf{l} \cdot \mathbf{n}) L_d$
 - k_d : diffuse reflection coefficient
 - L_d : intensity of light source
 - $\mathbf{n}$: unit normal vector of surface
 - $\mathbf{l}$: unit light direction vector
- attenuation form : (d)
 - a, b, c : constants

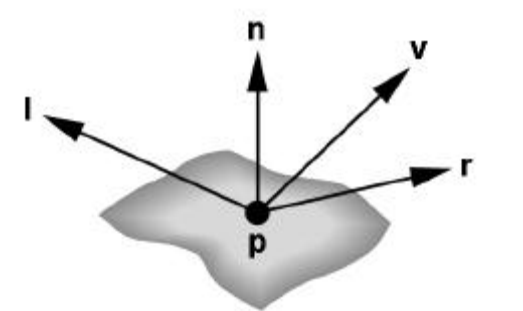
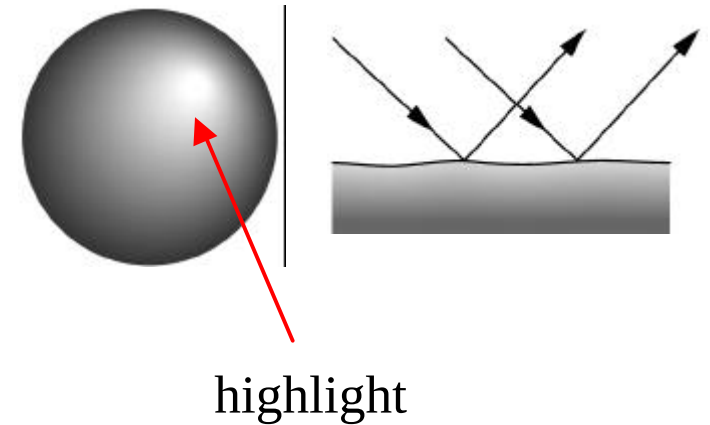
$$I_d = \frac{k_d}{a + bd + cd^2} (\mathbf{l} \cdot \mathbf{n}) L_d$$



Specular Reflection

- highlight
 - : perfect specular reflection
- $I_s = k_s \cos^\alpha \phi L_s = k_s (\mathbf{r} \cdot \mathbf{v})^\alpha L_s$
 - k_s : specular reflection coefficient
 - L_s : intensity of light source
 - $\mathbf{r}$: unit reflection vector
 - $\mathbf{v}$: unit view direction vector
 - α : shininess coefficient
- attenuation form : (d)
 - a, b, c : constants

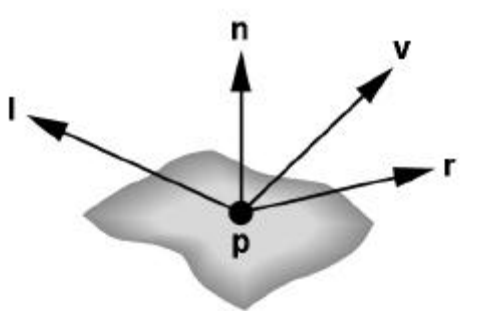
$$I_s = \frac{k_s}{a + bd + cd^2} (\mathbf{r} \cdot \mathbf{v})^a L_s$$



Phong Reflection Model

- Phong
 - ambient, diffuse, specular term

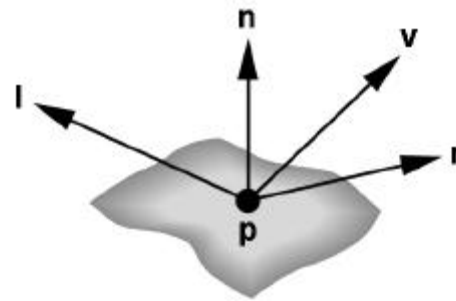
$$\begin{aligned} I &= I_a + I_d + I_s \\ &= k_a L_a + \frac{k_d}{a + bd + cd^2} (\mathbf{l} \cdot \mathbf{n}) L_d + \frac{k_s}{a + bd + cd^2} (\mathbf{r} \cdot \mathbf{v})^a L_s \\ &= \frac{1}{a + bd + cd^2} (k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a) + k_a L_a \end{aligned}$$



6.4 Computation of Vectors

Normal Vector

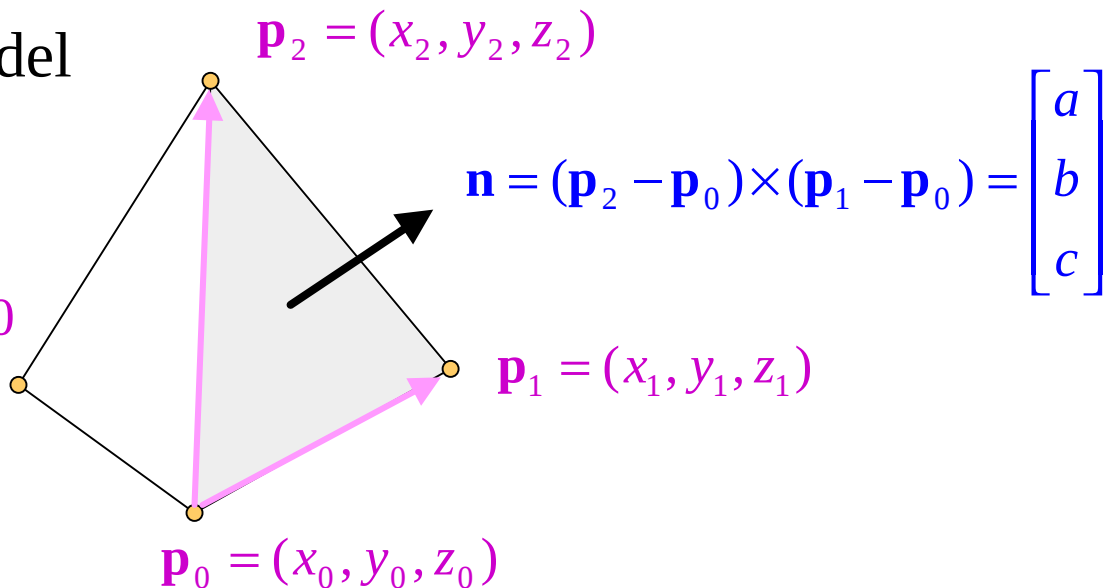
- Phong shading model



- $\mathbf{n}$: surface normal vector
 - polygonal model

surface equation

$$ax + by + cz + d = 0$$

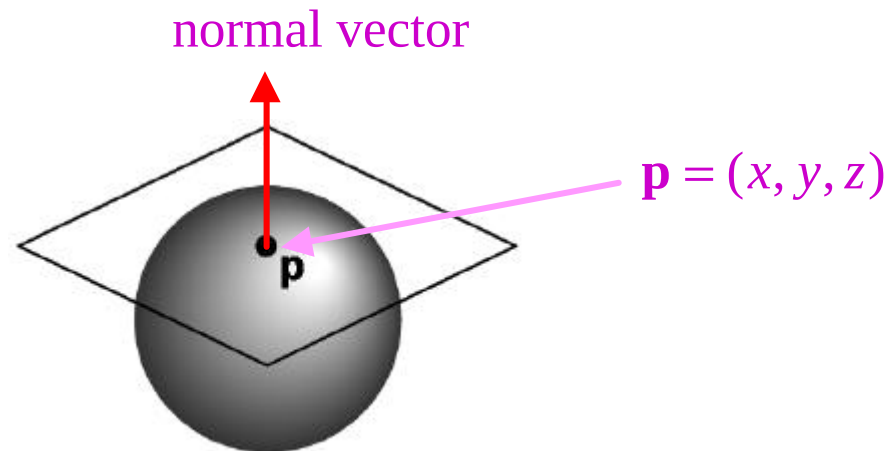


Normal Vector

- implicit equation :
 - : unit sphere

$$f(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$$

$$\mathbf{n} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$



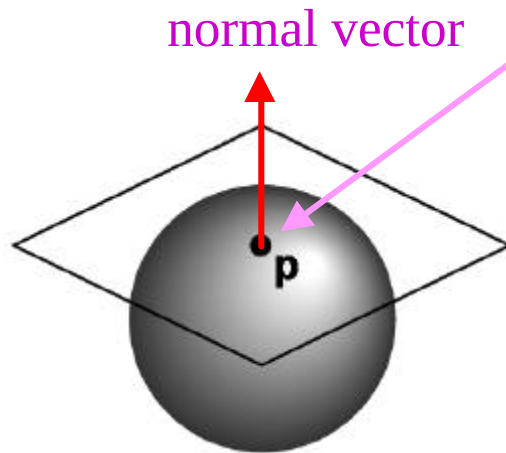
Normal Vector

- parametric equation :

$$x(u, v) = \cos u \sin v$$

$$y(u, v) = \cos u \cos v$$

$$z(u, v) = \sin u$$



$$\mathbf{p}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix}$$

$$\frac{\partial \mathbf{p}}{\partial u} = \begin{bmatrix} \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial u} \\ \frac{\partial z}{\partial u} \end{bmatrix}, \quad \frac{\partial \mathbf{p}}{\partial v} = \begin{bmatrix} \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial v} \end{bmatrix}$$

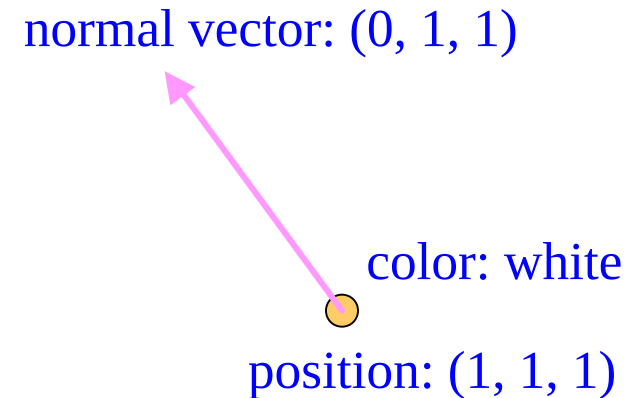
$$\mathbf{n} = \frac{\partial \mathbf{p}}{\partial u} \times \frac{\partial \mathbf{p}}{\partial v}$$

Normal Vector

- in OpenGL
 - void `glNormal3f`(float x, float y, float z);
 - void `glNormal3fv`(float v[3]);
 - use the normal vector for a vertex

- example

```
glBegin(GL_QUADS);  
    glColor3f(1.0, 1.0, 1.0);  
    glNormal3f(0.0, 1.0, 1.0);  
    glVertex3f(1.0, 1.0, 1.0);  
    ...  
glEnd(GL_QUADS);
```



Reflection Vector

- light direction **$\mathbf{l}$** :
- reflection vector **$\mathbf{r}$** :
- **$\mathbf{l}$, $\mathbf{n}$, $\mathbf{r}$** normalize

$$|\mathbf{l}| = |\mathbf{n}| = |\mathbf{r}| = 1$$

$$q_i = q_r \Rightarrow \cos q_i = \cos q_r$$

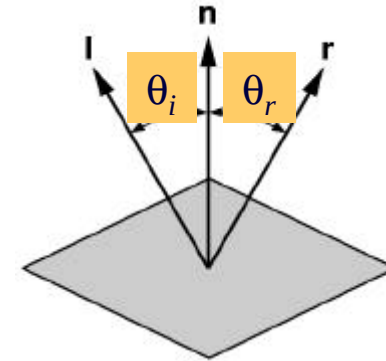
$$\cos q_i = \mathbf{l} \cdot \mathbf{n} = \cos q_r = \mathbf{n} \cdot \mathbf{r}$$

$$\text{let } \mathbf{r} = a \mathbf{l} + b \mathbf{n}$$

$$\mathbf{n} \cdot \mathbf{r} = a \mathbf{l} \cdot \mathbf{n} + b = \mathbf{l} \cdot \mathbf{n}$$

$$\mathbf{r} \cdot \mathbf{r} = a^2 + 2ab \mathbf{l} \cdot \mathbf{n} + b^2$$

$$\mathbf{r} = 2(\mathbf{l} \cdot \mathbf{n})\mathbf{n} - \mathbf{l}$$



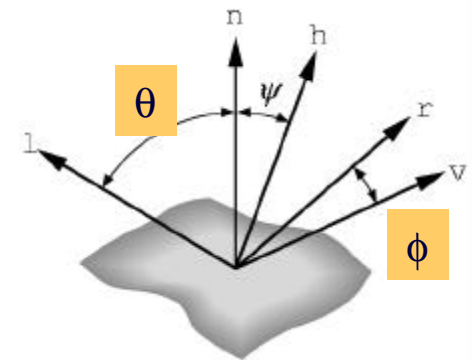
Halfway Vector

- J. Blinn, “[Models of Light Reflection For Computer Synthesized Pictures](#)”, *SIGGRAPH*’77, 192–198, (1977).
- specular reflection : $I_s = k_s \cos^\alpha \phi L_s = k_s (\mathbf{r} \cdot \mathbf{v})^\alpha L_s$
- halfway vector $\mathbf{h}$: ...

$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{|\mathbf{l} + \mathbf{v}|}$$

$$\mathbf{r} \cdot \mathbf{v} = \cos f = \cos 2y$$

$$\mathbf{n} \cdot \mathbf{h} = \cos y < \cos f$$



- $(\mathbf{r} \cdot \mathbf{v})^\alpha \approx (\mathbf{n} \cdot \mathbf{h})^{\alpha'}$, α'
 - approximation ,
 - 가
 -

Transmitted Light

- (refraction)

– ,

- Snell's law :

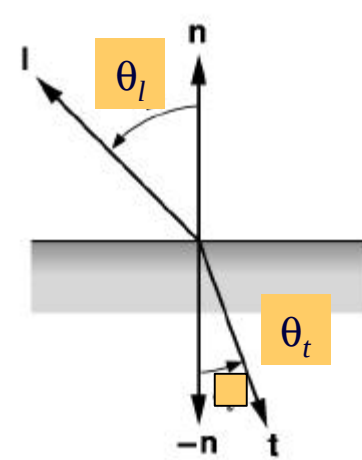
$$\frac{\sin q_l}{\sin q_t} = \frac{h_t}{h_l} = h$$

- η_l, η_t : index of refraction ()

$$\cos q_t = \sqrt{1 - \frac{1}{h^2} (1 - \cos^2 q_l)}$$

$$\mathbf{t} = a \mathbf{n} + b \mathbf{l}$$

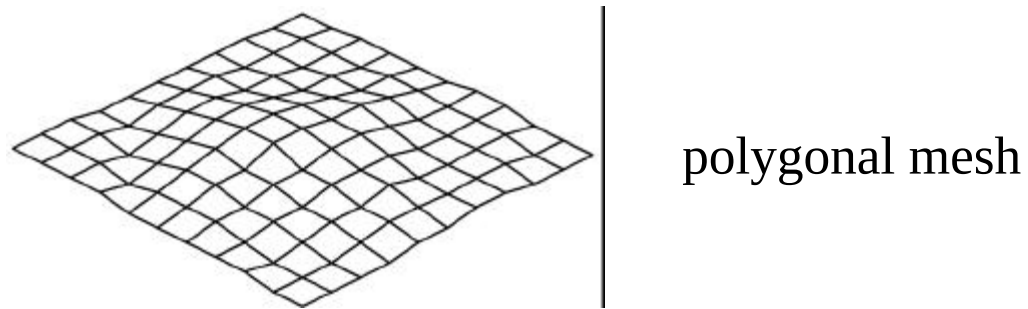
$$\mathbf{t} = -\frac{1}{h} \mathbf{l} - \left(\cos q_t - \frac{1}{h} \cos q_l \right) \mathbf{n}$$



6.5 Polygonal Shading

Polygon Shading

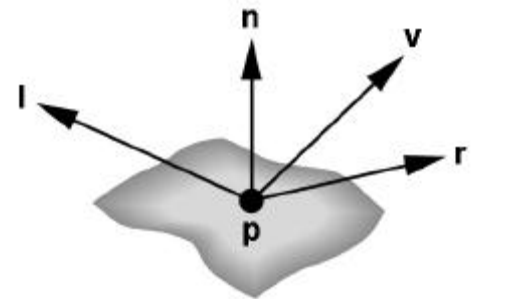
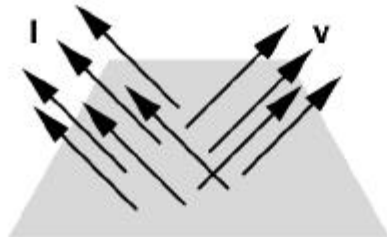
- polygonal object shading
 - 71 polygonal object



- 71 ,
 - flat shading
 - Gouraud shading
 - Phong shading

Flat Shading

- = constant shading
 - 가 1: flat polygon $\mathbf{n} = \text{constant}$
 - 가 2: distant viewer $\mathbf{v} = \text{constant}$
 - 가 3: distant light $\mathbf{l} = \text{constant}, \mathbf{r} = \text{constant}$
 - , polygon
shading 가

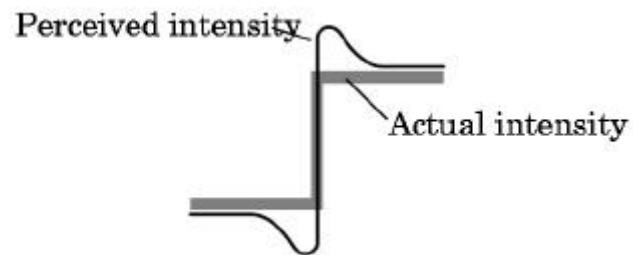
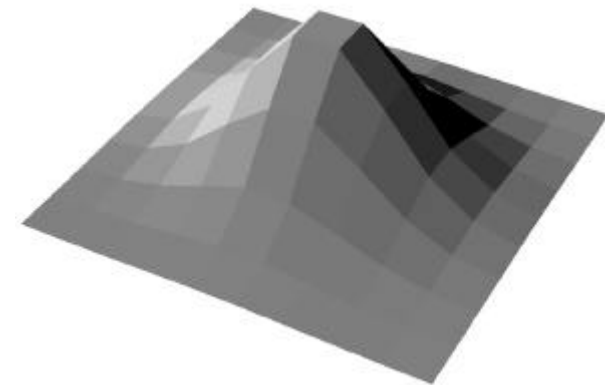


Flat Shading

- shading Phong model

$$I = \frac{1}{a + bd + cd^2} (k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a) + k_a L_a$$

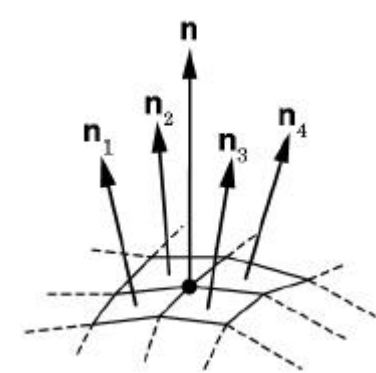
- in OpenGL
`glShadeModel(GL_FLAT);`
- : Mach band effect
 - polygon



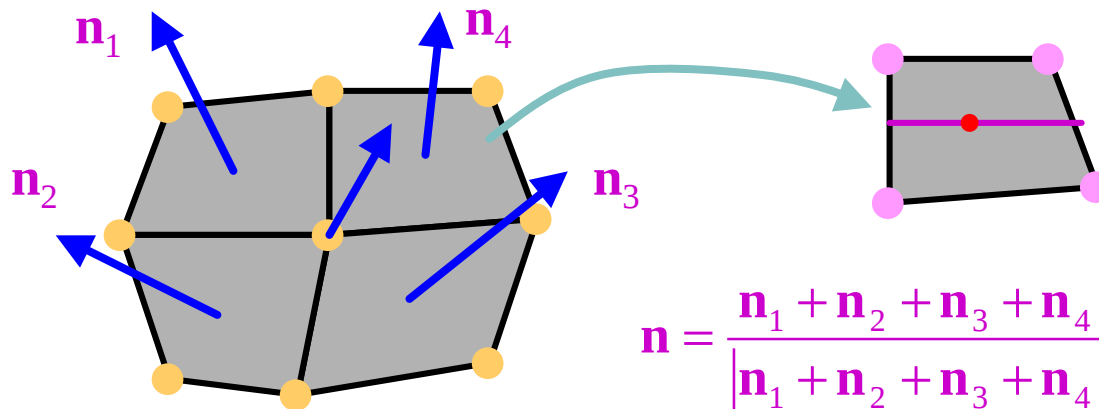
Gouraud Shading

- = interpolative shading
- Mach band effect
 - vertex normal
 - face normal

?



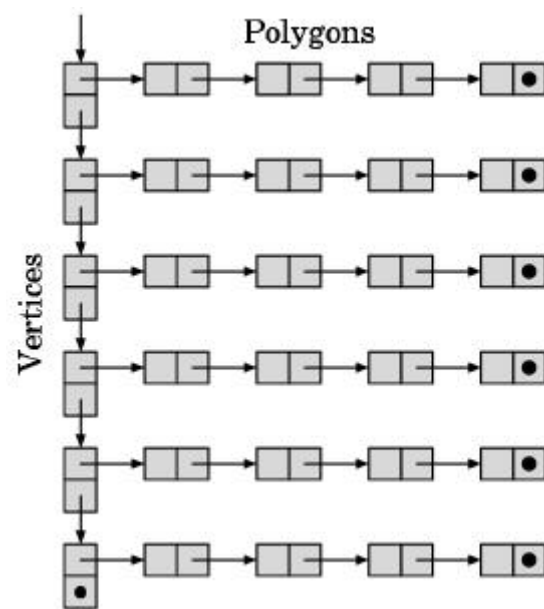
- face shading
- vertex shading
- bi-linear interpolation



bi-linear interpolation

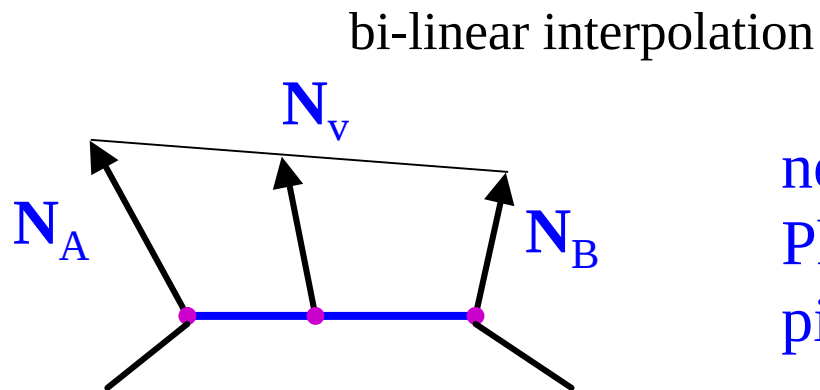
Gouraud Shading

- in OpenGL
`glShadeModel(GL_SMOOTH);`
- - vertex normal data structure
 - , linked list



Phong Shading

- Gouraud shading
 - vertex shading interpolate
 - highlight 가
- Phong shading : normal vector interpolate
 - shading pixel
 -



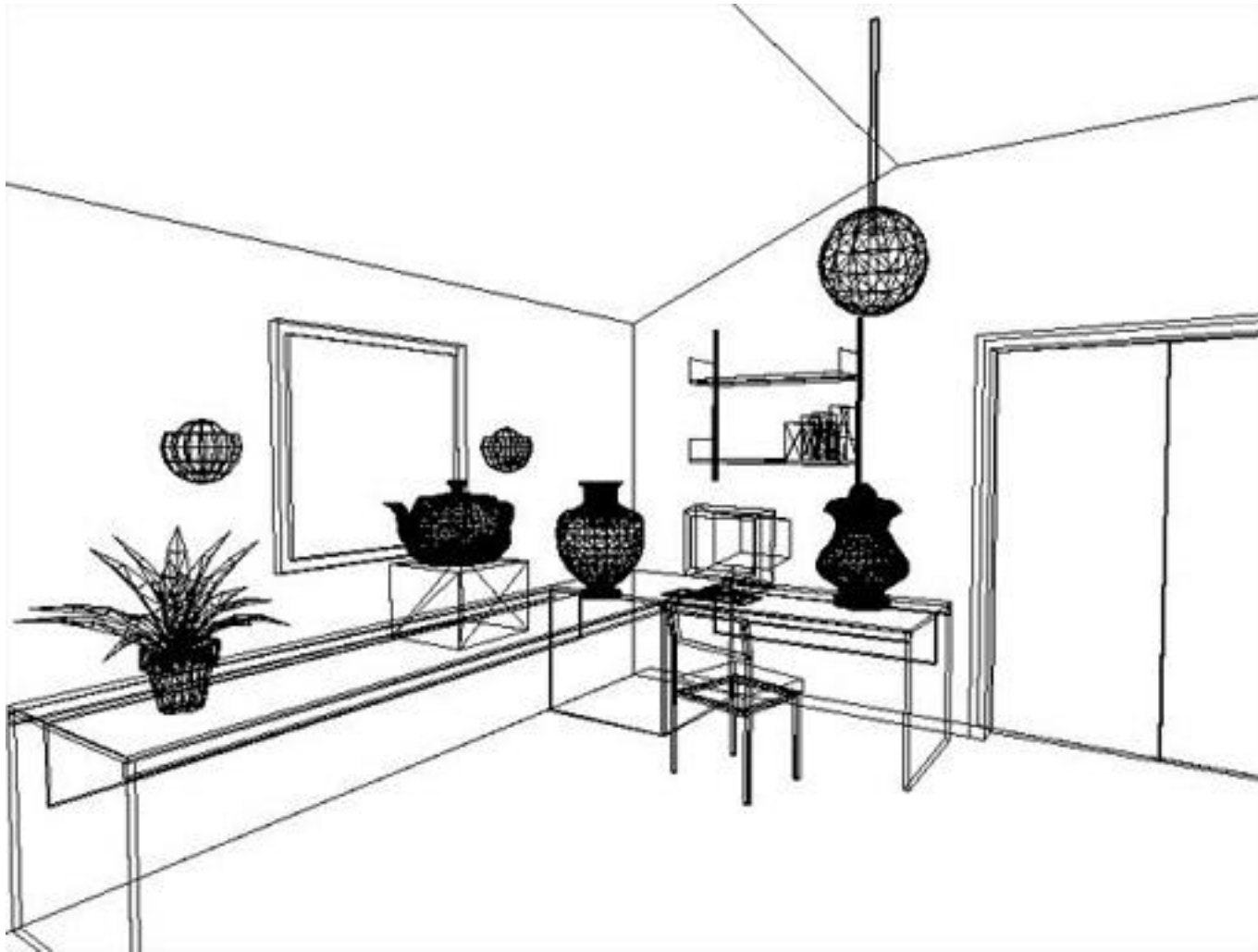
normal vector
Phong model
pixel

가

Rendering Options

- graphics package
 - quality vs. speed trade-off
- wireframe edge
- flat-shading face
- Gouraud shading
- Phong shading
- mixed shading
 - non-specular object : Gouraud shading
 - specular object : Phong shading

Wire-frame



Ambient Illumination Only



Flat-Shading



Gouraud Shading



Phong Shading



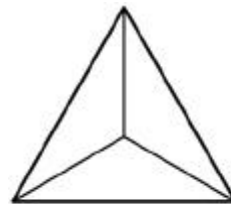
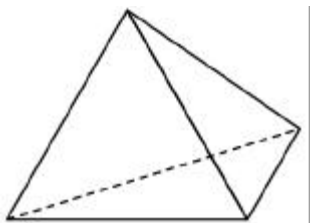
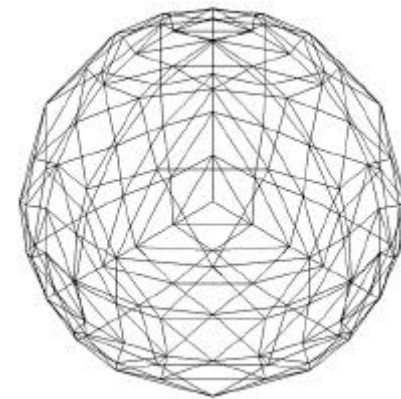
6.6 Approximation of a Sphere

Unit Sphere

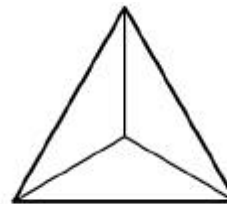
- implicit equation $\Rightarrow$ polygonal approximation

$$f(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$$

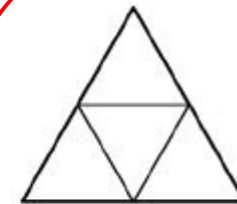
- recursion
 - : tetrahedron
 - face subdivision



(a)



(b)

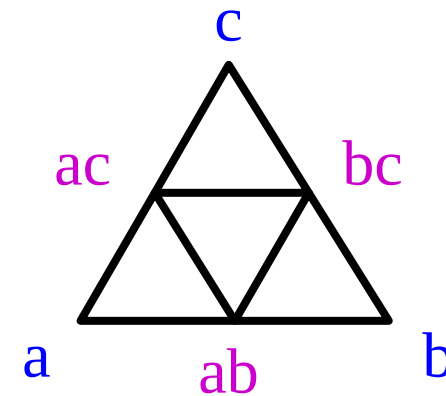


(c)

Unit Sphere

- pseudo code

```
void triangle(vertex a, vertex b, vertex c, int level) {  
    if (level >= limit) {  
        draw_triangle(a, b, c);  
        return;  
    }  
    vertex ab = normalize( (a + b) / 2 );  
    vertex bc = normalize( (b + c) / 2 );  
    vertex ac = normalize( (a + c) / 2 );  
    triangle(a, ab, ac);  
    triangle(ab, b, bc);  
    triangle(c, ac, bc);  
    triangle(ab, bc, ac);  
}
```



6.7 Light Sources in OpenGL

Light Sources in OpenGL

- Phong shading model

$$\begin{aligned} I &= I_a + I_d + I_s \\ &= k_a L_a + \frac{k_d}{a + bd + cd^2} (\mathbf{l} \cdot \mathbf{n}) L_d + \frac{k_s}{a + bd + cd^2} (\mathbf{r} \cdot \mathbf{v})^a L_s \\ &= \frac{1}{a + bd + cd^2} (k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a) + k_a L_a \end{aligned}$$

- at least 8 light sources
 - GL_LIGHT0, ..., GL_LIGHT7

```
glEnable(GL_LIGHTING);           // light source
glEnable(GL_LIGHT0);              // light source #0 on
```

Light Sources in OpenGL

- light source

`void glLightf(GLenum light, GLenum name, GLfloat value);`

`void glLightfv(GLenum light, GLenum name, GLfloat* values);`

- light source position (or direction)

`glLightfv(GL_LIGHT0, GL_POSITION, light0_pos);`

- point light source, spotlight : position

`GLfloat light0_pos[ ] = { 1.0, 2.0, 3.0, 1.0 };`

- distant light source : direction vector

`GLfloat light0_pos[ ] = { 1.0, 2.0, 3.0, 0.0 };`

$$I = \frac{1}{a + bd + cd^2} \left(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a \right) + k_a L_a$$

Light Sources in OpenGL

- the amount of ambient, diffuse, specular light
GLfloat ambient_0[] = {1.0, 0.0, 0.0, 1.0}; // RGBA rep.
glLightfv(GL_LIGHT0, GL_AMBIENT, ambient_0);
glLightfv(GL_LIGHT0, GL_DIFFUSE, diffuse_0);
glLightfv(GL_LIGHT0, GL_SPECULAR, specular_0);
- global ambient light (if necessary)
glLightModelfv(GL_LIGHT_MODEL_AMBIENT, global_ambient);

$$I = \frac{1}{a + bd + cd^2} \left(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a \right) + k_a L_a$$

Light Sources in OpenGL

- distance attenuation model

`glLightf(GL_LIGHT0, GL_CONSTANT_ATTENUATION, a);`

`glLightf(GL_LIGHT0, GL_LINEAR_ATTENUATION, b);`

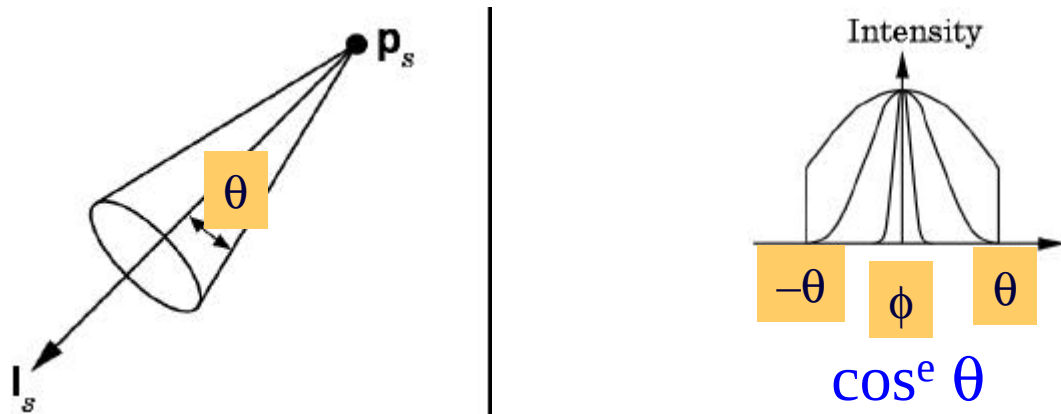
`glLightf(GL_LIGHT0, GL_QUADRATIC_ATTENUATION, c);`

$$I = \frac{1}{a + bd + cd^2} (k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a) + k_a L_a$$

Light Sources in OpenGL

- spot light

```
glLightf(GL_LIGHT0, GL_SPOT_CUTOFF, 30.0F);  
glLightf(GL_LIGHT0, GL_SPOT_EXPONENT, e);  
glLightfv(GL_LIGHT0, GL_SPOT_DIRECTION, vector);
```



Light Sources in OpenGL

- viewer location

- default: infinite position (COP)

- COP 가

- , shading v

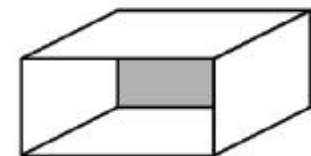
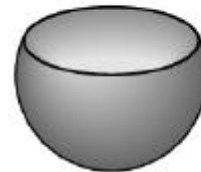
```
glLightModeli(GL_LIGHT_MODEL_LOCAL_VIEWER, GL_TRUE);
```

$$I = \frac{1}{a + bd + cd^2} (k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a) + k_a L_a$$

- two-side shading

- back face shading

```
glLightModeli(GL_LIGHT_MODEL_TWO_SIDED, GL_TRUE);
```



6.8 Specification of Materials in OpenGL

Materials in OpenGL

- material functions

`void glMaterialf(GLenum face, GLenum name, GLfloat value);`

– face 가

– face = GL_FRONT, GL_BACK, GL_FRONT_AND_BACK

`GLfloat ambient[ ] = {0.2, 0.2, 0.2, 1.0}`

`glMaterialfv(GL_FRONT_AND_BACK, GL_AMBIENT, ambient);`

`glMaterialfv(GL_FRONT, GL_DIFFUSE, diffuse);`

`glMaterialfv(GL_BACK, GL_SPECULAR, specular);`

$$I = \frac{1}{a + bd + cd^2} \left(\underbrace{k_d}_{\text{red oval}} L_d (\mathbf{l} \cdot \mathbf{n}) + \underbrace{k_s}_{\text{red oval}} L_s (\mathbf{r} \cdot \mathbf{v})^a \right) + \underbrace{k_a}_{\text{red oval}} L_a$$

Materials in OpenGL

- shininess

```
glMaterialf(GL_BACK, GL_SHININESS, 100.0);
```

$$I = \frac{1}{a + bd + cd^2} \left(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a \right) + k_a L_a$$

- self-emission

– surface 가

```
GLfloat emission[ ] = {0.0, 0.3, 0.3, 1.0};
```

```
glMaterialf(GL_FRONT_AND_BACK, GL_EMISSION, emission);
```

$$I = \frac{1}{a + bd + cd^2} \left(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^a \right) + k_a L_a + emission$$

Materials in OpenGL

- color tracking
 - set the materials parameters by tracking the colors set by glColor*()

```
glEnable(GL_COLOR_MATERIAL);
```

```
glColorMaterial(GL_FRONT, GL_AMBIENT_AND_DIFFUSE);
```

```
glColor4fv(color);
```

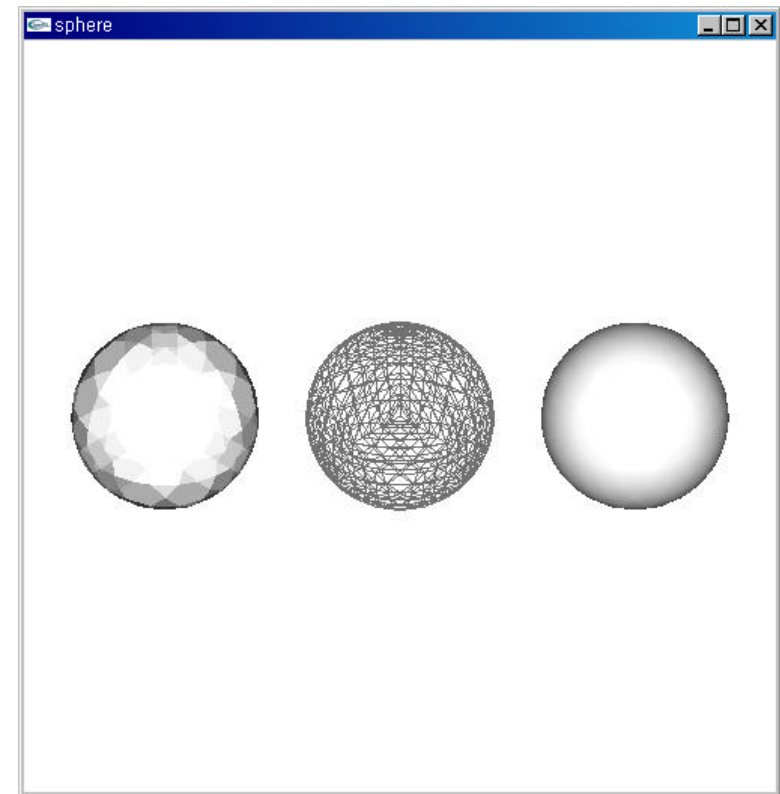


```
glMaterialfv(GL_FRONT, GL_AMBIENT, color);  
glMaterialfv(GL_FRONT, GL_DIFFUSE, color);
```

6.9 Shading of the Sphere Model

Unit Sphere Shading

- 2가지 shading model 가
 - flat shading
`glShadeModel(GL_FLAT);`
 - Gouraud shading
`glShadeModel(GL_SMOOTH);`
- - sphere.c

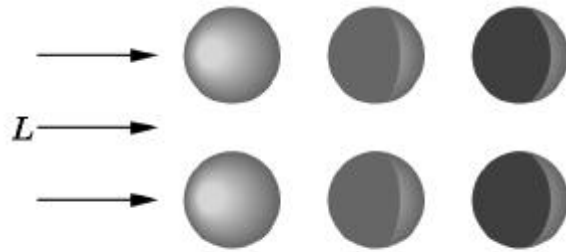


6.10 Global Rendering

Local and Global Illumination

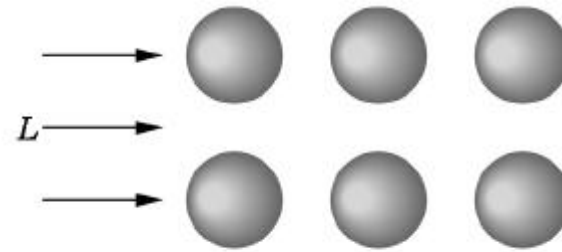
- local illumination : Phong shading model

– , 가



(a)

global illumination



(b)

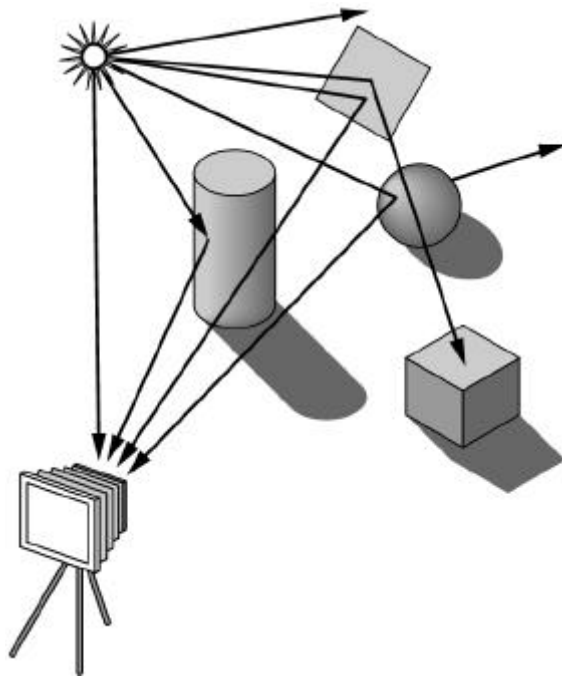
local illumination

- global illumination
 - Ray Tracing : specular, ,
 - Radiosity : diffuse 가

Ray Tracing

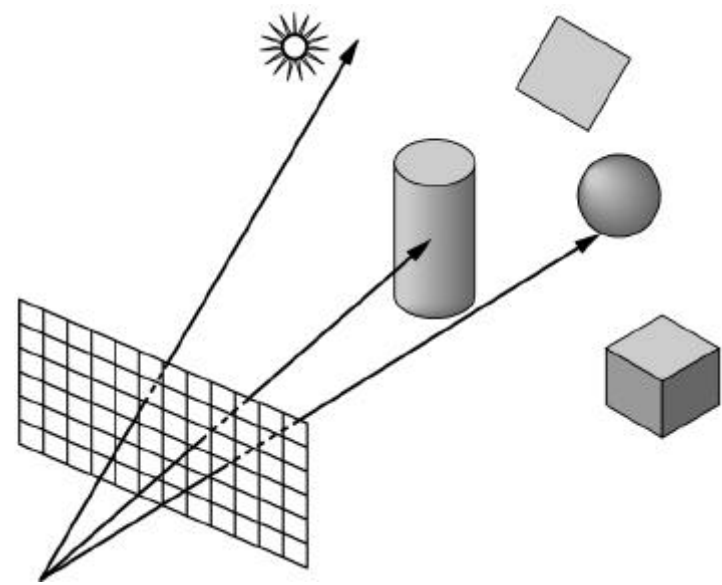
- forward ray tracing :

– $\Rightarrow \Rightarrow$



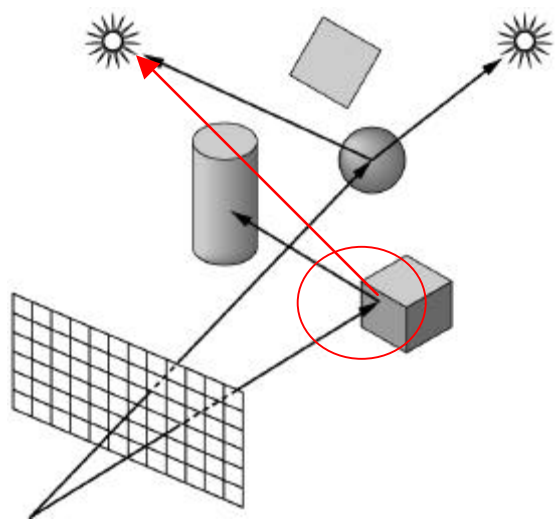
- backward ray tracing

– ,
– $\Rightarrow \Rightarrow \Rightarrow$



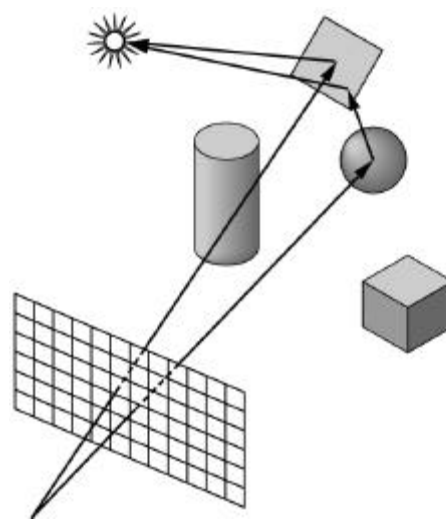
Ray Tracing

- A. Glassner, *An Introduction to Ray Tracing*, Academic Press, (1989).
- ray trace 가 ,



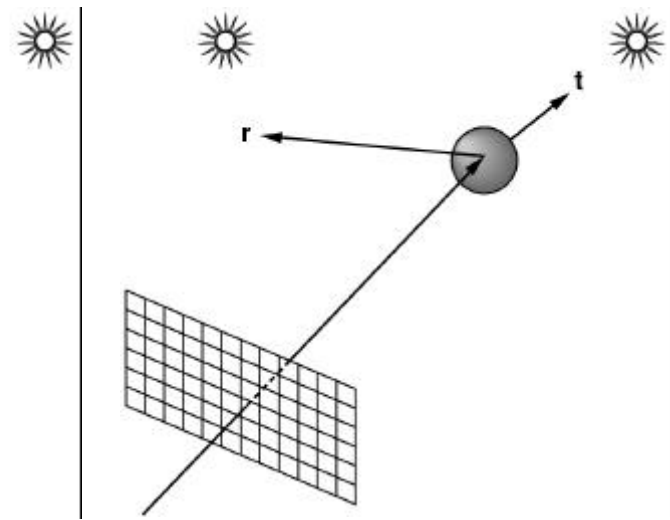
shadow rays

가



mirror effect

가

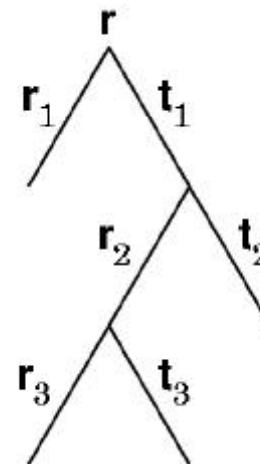
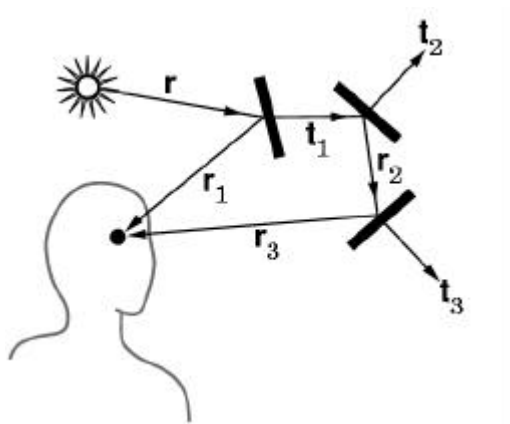


refraction & reflection

가

Ray Tracing

- screen pixel 가 ray
 - construct a binary ray-tracing tree
 - left branch : reflected ray path
 - right branch : transmitted ray path
 - limit depth
 - tree , pixel

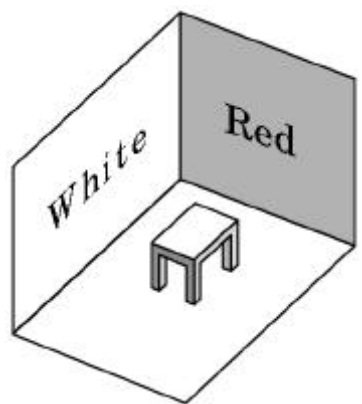


Ray Tracing



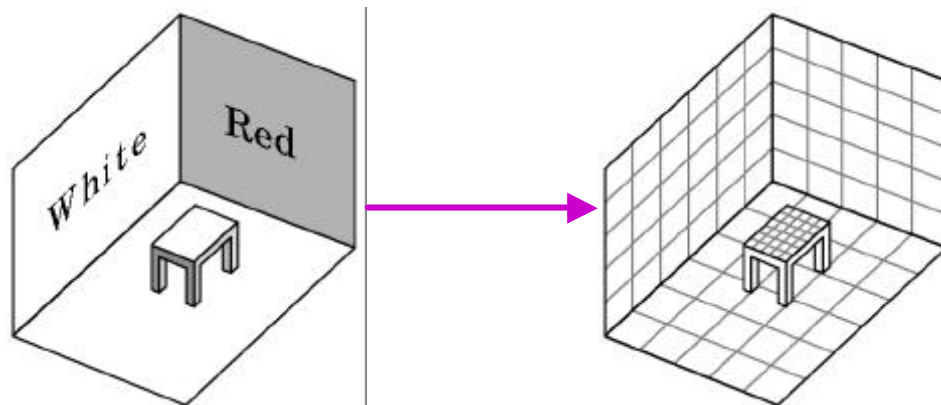
Radiosity

- C. Goral, et al., “[Modelling the Interaction of Light Between Diffuse Surfaces](#)”, *SIGGRAPH*’84, 212–222, (1984).
- Basic idea
 - [thermal-engineering models](#) for the emission and reflection of radiation
 - [conservation of light energy](#) in a closed environment
 -
 - [energy equilibrium](#)



Radiosity

- real diffuse effect ?
 - surface light interaction modeling
 - surface
- patch :
 - surface patch
 - patch diffuse term
 - form factor : patch 1 patch 가 energy

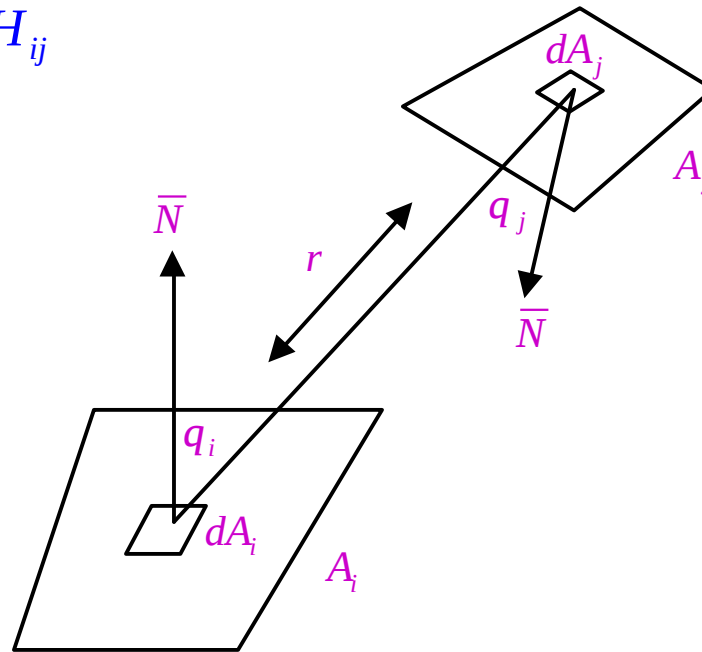


Radiosity

- form factors

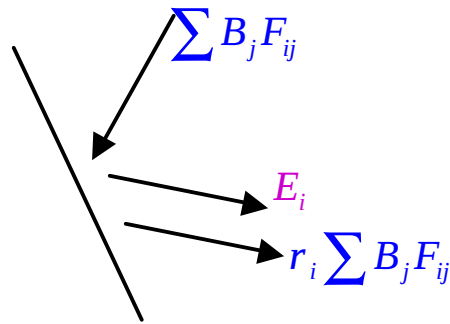
$$F_{ij} = 1/A_i \int \int_{A_i A_j} (\cos q_i \cos q_j / (pr^2)) H_{ij} dA_j dA_i$$

$$\approx \int_{A_j} \frac{\cos q_i \cos q_j}{pr^2} dA_j H_{ij}$$



Radiosity

- energy equilibrium equation



$$B_i A_i = E_i A_i + r_i \sum_{j=1}^n B_j F_{ji} A_j$$

$$F_{ij} A_i = F_{ji} A_j$$

$$\therefore B_i = E_i + r_i \sum_j B_j F_{ij}$$

- radiosity B_j : the rate of energy leaving the surface j
- emission E_j : the rate of energy emitted from surface j
- reflectivity r_j : the fraction of the light incident on a surface j which is reflected back into the environment
- form-fraction F_{ij} : the fraction of energy leaving the surface i and landing on surface j

Radiosity



Global Illumination

- ray tracing : specular effect
- radiosity : diffuse effect
- hybrid approach : 2

- ?

— !

— video card level

가

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