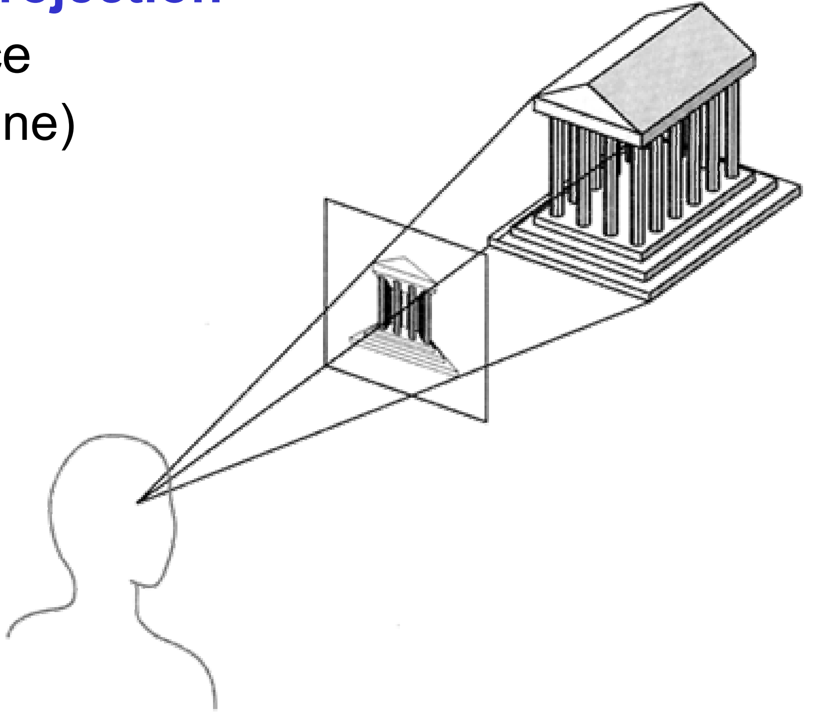
Projection: Mapping 3-D to 2-D

Our scene models are in 3-D space and images are 2-D

- so we need some way of projecting 3-D to 2-D

The fundamental approach: planar projection

- first, we define a plane in 3-D space
  - this is the image plane (or film plane)
- then project scene onto this plane
- and map to the window viewport



Need to address two basic issues

- how to define plane
- how to define mapping onto plane

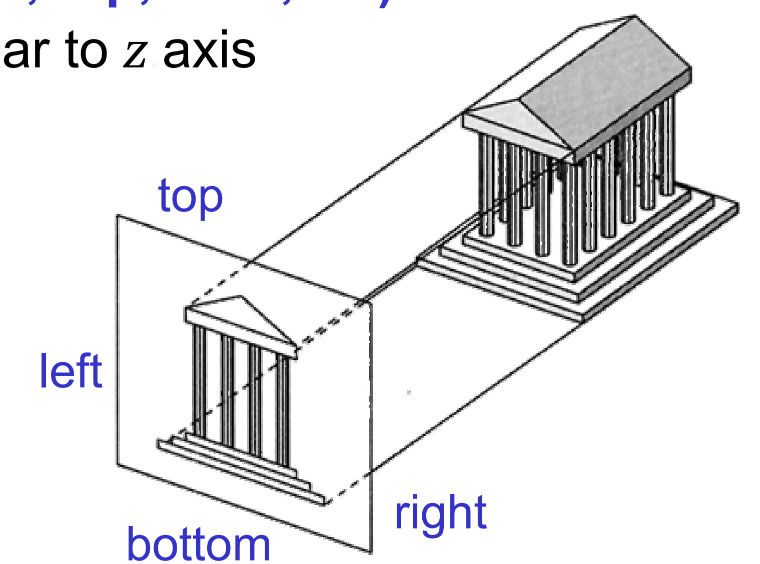
Orthographic Projection

Arguably the simplest projection

- image plane is perpendicular to one of the coordinate axes
- project onto plane by dropping that coordinate
- $(x, y, z) \to (x, y)$ or $\to (x, z)$ or $\to (y, z)$

OpenGL — glOrtho(left, right, bottom, top, near, far)

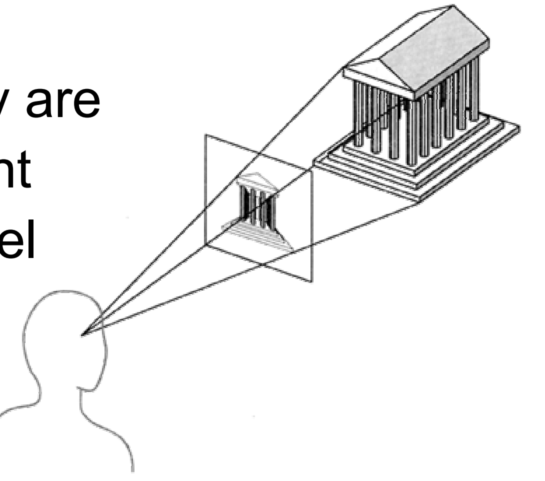
- assumes image plane perpendicular to z axis
  - in other words, it's the xy-plane
- projects points $(x, y, z) \to (x, y)$
- also defines viewport mapping
  - defines rectangle on xy-plane
  - this gets mapped to window



Perspective Projection

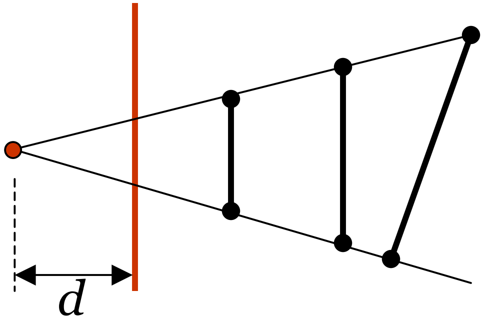
But we naturally see things in perspective

- objects appear smaller the farther away they are
- lenses bend (and hence focus) incoming light
- in orthographic projection, all rays are parallel

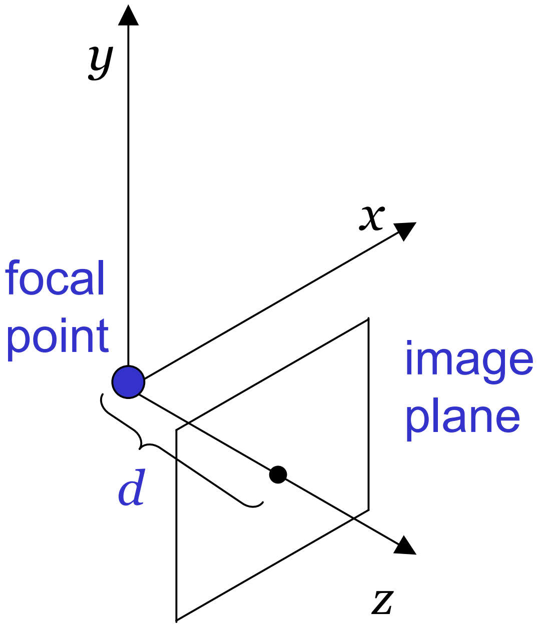


We've been using pinhole camera models

- draw rays thru focal point and points on object
- some of these lines will intersect the image plane
- this defines our projection into 2-D
- all points along a ray project to same point
- can project lines by projecting endpoints



The Canonical Camera Configuration



Want to derive perspective transformation

- in particular, a matrix representation

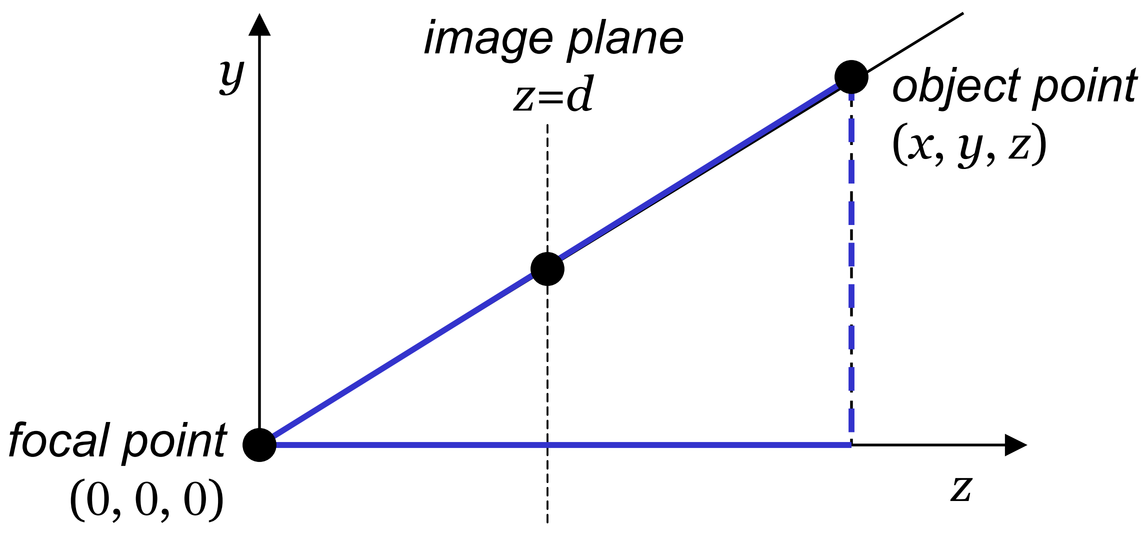
First, we fix a canonical camera

- focal point at origin
- looking along z axis
- image plane parallel to xy plane
- located distance d from origin
  - called the focal length

Effect of Perspective Projection on Points

We project points thru the line connecting them to the focal point

- given a point, we want to know where this line hits the image plane



Effect of Perspective Projection on Points

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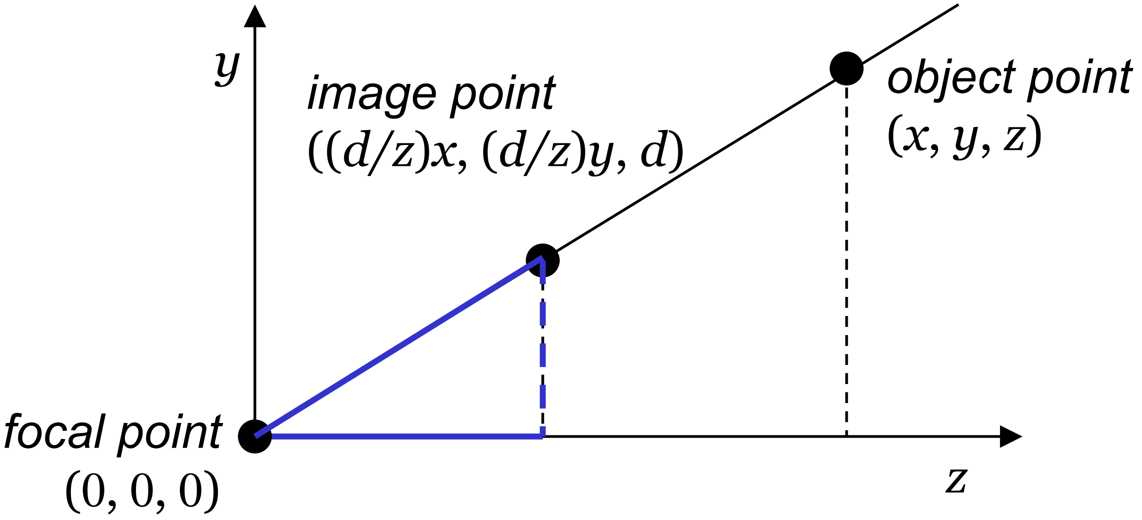
Can easily compute this using similar triangles



Perspective Projection as a Transformation

This homogeneous matrix performs perspective projection

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

It’s operation on any given point is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

Perspective Projection as a Transformation

This homogeneous matrix performs perspective projection

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

And when we do the homogeneous division

- we get exactly the point we want
- only keep x and y coordinates

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \Rightarrow \begin{bmatrix} \left(\frac{d}{z}\right)x \\ \left(\frac{d}{z}\right)y \\ d \\ 1 \end{bmatrix}$$

Completing the Projection

The image plane itself is infinite

- must map a rectangular region of it to the viewport
- defined by (*left, right, top, bottom*) coordinates

We also customarily define near & far clipping planes

- these are expressed as distances from the viewpoint
- they should always be positive
- nothing nearer than *near* will be drawn
  - don't want to draw things behind the image plane
- nothing further than *far* will be drawn
- distance far-near should be small
  - use fixed precision numbers to represent depth between them

OpenGL — glFrustum(left, right, bottom, top, near, far)

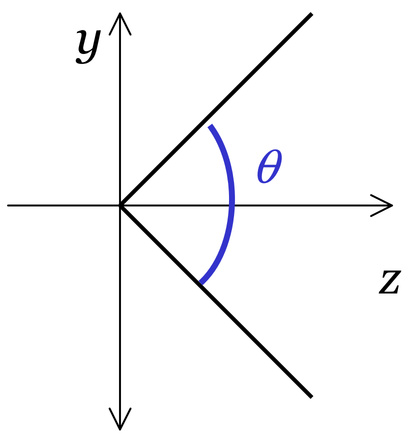
More Convenient Perspective Specification

Could always use glFrustum(left, right, bottom, top, near, far)

- this is certainly sufficient
- but it's inconvenient

Generally want to use: gluPerspective(fovy, aspect, near, far)

- viewport is always centered about z axis
- specifies the field of view along the y axis
  - the angle θ made by the sides of the frustum
- and the aspect ratio of the viewport
  - this is just (*width / height)*



Viewing Volumes

The sides of the viewport define an infinite pyramid

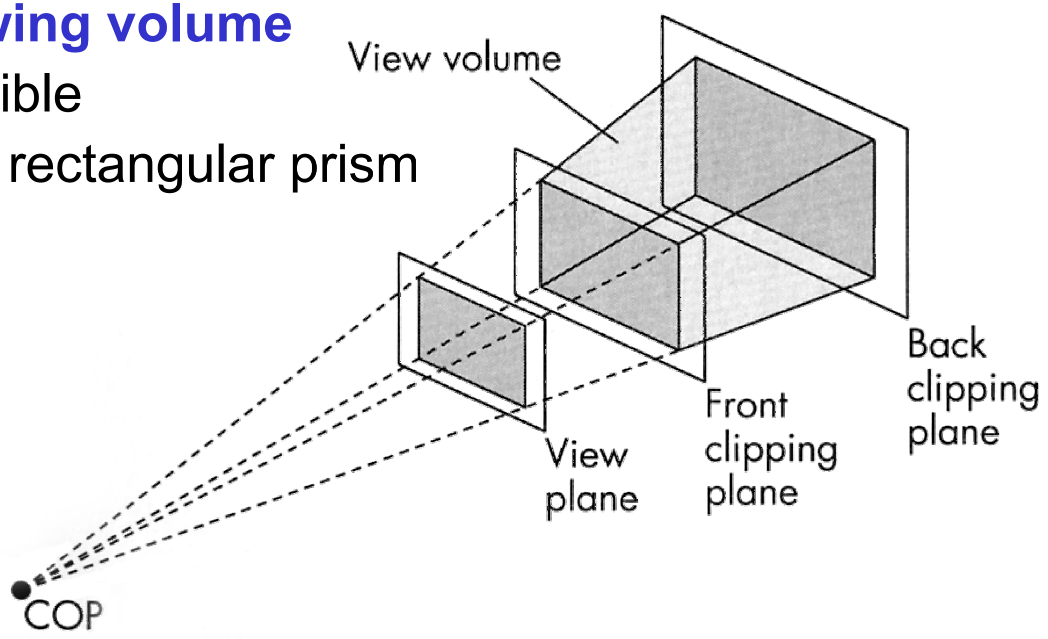
- focal point at apex, extending outward through space

Adding in the clipping planes, we get a truncated pyramid

- this is called a frustum

Can think of this as the viewing volume

- nothing outside of it is visible
- projection warps this to a rectangular prism



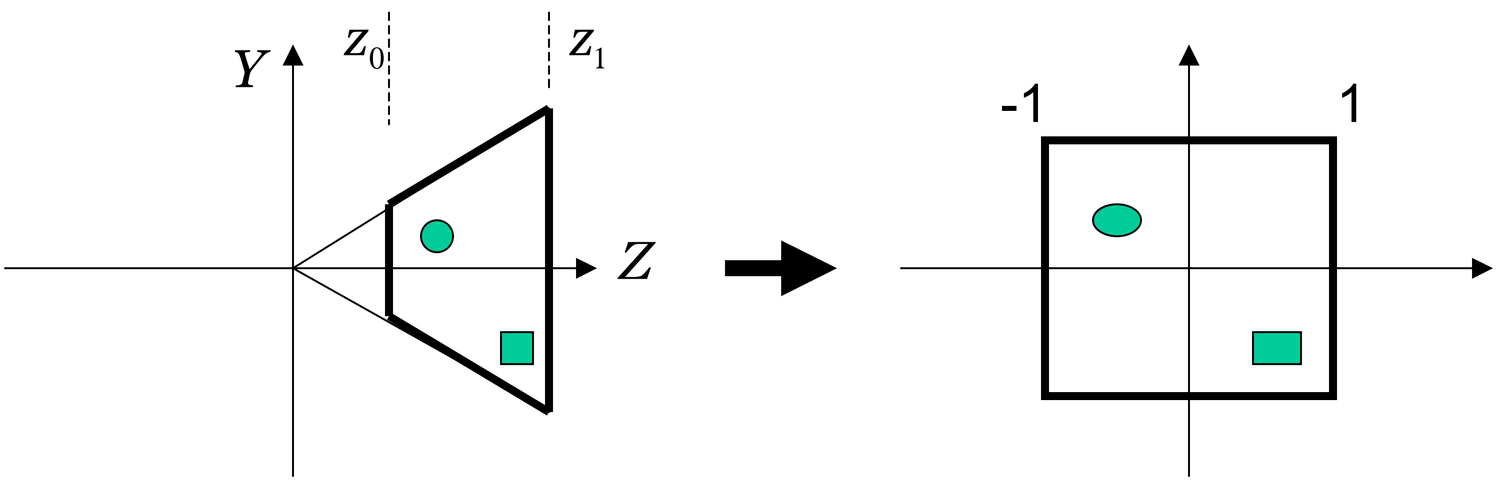
Example: gluPerspective in Action

$$\mathbf{P} = \begin{bmatrix} \cot(\frac{\theta}{2})/\alpha & 0 & 0 & 0 \\ 0 & \cot(\frac{\theta}{2}) & 0 & 0 \\ 0 & 0 & \frac{z_1 + z_0}{z_0 - z_1} & \frac{2 z_1 z_0}{z_0 - z_1} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

θ : field of view (fovy)

α : aspect ratio

z_0, z_1 : zNear and zFar



We Need More General Cameras

So far, we've assumed a "canonical" camera configuration

- focal point at the origin
- image plane parallel to xy-plane

This is pretty limited, we want greater flexibility

- deriving general projection matrices is painful
- but we can transform world so camera is canonical
- typically called the viewing transformation

Naturally, there are several ways of setting this up

- we'll focus on the OpenGL supported mechanism
- the one in the book is gratuitously complex

Specifying General Camera Configurations

First, we want to allow focal point to be anywhere in space

- call this position *lookFrom*, or just *from*

Next, we need to specify the orientation of the camera

- define what it's pointing at: *lookAt*
  - *lookAt*–*lookFrom* will define the axis of projection
- define vertical axis of image: *vUp*
  - essentially a twist parameter about the *lookAt* axis

Converting Camera to Canonical Form

Our camera is parameterized by three vectors

- *lookFrom*, *lookAt*, and *vUp*

We want to transform into canonical camera position

1. translate *lookFrom* to the origin — translate by –*lookFrom*
2. rotate *lookAt*–*lookFrom* to the z axis

   Axis: $\mathbf{u} = (\mathit{lookAt} - \mathit{lookFrom}) \times \mathbf{z}$

   Angle: $\theta = \sin^{-1}(\|\mathbf{u}\| / L)$ where $L = \|\mathit{lookAt} - \mathit{lookFrom}\| \, \|\mathbf{z}\|$

3. rotate about z so that *vUp* lies inside the y-z plane

OpenGL Transformation Matrices

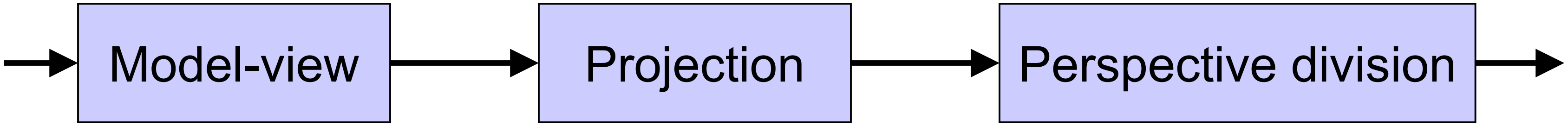
OpenGL maintains two different matrices

- one to hold the camera projection matrix
- and one to hold everything else
- select “current matrix” with glMatrixMode(*which*)
  - *which* is GL_MODELVIEW or GL_PROJECTION

glFrustum() and friends multiply the current matrix

- just like glTranslate(), glScale(), glRotate()

Vertices are transformed in the following manner



OpenGL Viewing Transformations

Specify camera configuration with

gluLookAt(ex, ey, ez, ax, ay, az, ux, uy, uz)

These are our three camera vectors

- *lookFrom* (ex, ey, ez)
- *lookAt* (ax, ay, az)
- *vUp* (ux, uy, uz)

Typical Transformation Setup:

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
gluPerspective(fovy, aspect, zNear, zFar);

glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookat(ex, ey, ez, ax, ay, az, 0, 1, 0);
```

Demo

See "Links" web page for link to OpenGL tutors

Next Time: Illumination & Shading

To make nice pictures, we need to shade surfaces

- how to simulate the interaction of light with a surface?
- in other words, how do we define its appearance?