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Discrete-Time $\mathrm{Geo}^X/\mathrm{G}/1$ Queue With Preemptive Resume Priority

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Abstract—This paper considers a $\mathrm{Geo}^X/\mathrm{G}/1$ queue with preemptive resume priority. Service times of messages of each priority class are i.i.d. according to a general distribution function that may differ between two classes. The analysis is based on the generating function technique and the supplementary variable method. We derive the joint system occupancy distributions at various observation instant and provide the analysis of the system time and the busy period.

Keywords—Discrete-time queue, Preemptive resume priority, System occupancy distribution, System time.

1. INTRODUCTION

Priority mechanisms are an invaluable scheduling method that allows messages of different classes to receive different quality of service (QoS). For this reason, the priority queue has received considerable attention in the literature. Two well-known priority disciplines in queueing literature are the *nonpreemptive* and the *preemptive* disciplines. The first was introduced by Cobham [1]. Under this rule, if a message of high priority arrives when a message of low priority is being served, it waits until the message in service completes its service. The second assumes that a message of high priority has a right of replacing the message of low priority from the server. Since messages of high priority interrupt the service of a message of low priority, the message of low priority on its re-entry may either resume its service from the preempted point, or repeat its service. The former is called the preemptive resume rule and the latter is called the preemptive repeat rule. Obviously under the resume rule, the service time of a message of low priority upon re-entry gets reduced by the amount of time the message has already spent in service.

Let us review some related papers. Many authors have studied priority queueing systems. Cobham [1], Lee [2], Machihara [3], and Sugahara *et al.* [4] have studied continuous-time priority queues. Numerous examples of discrete-time queues can be found operating within present computer and communication systems. The analysis of such discrete-time priority queues is therefore very important and numerous studies of it exist within the literature. Choi *et al.* [5,6], Hashida and Takahashi [7], Khamisy and Sidi [8], and Takine *et al.* [9] have studied discrete-time priority

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queues, where service times of messages of all classes are assumed to be constant and equal to the slot size. As a result, there is no distinction between the nonpreemptive and the preemptive priority systems. Sidi [10] and Sidi *et al.* [11] have analyzed the discrete-time structured input queue under nonpreemptive priority rule, where the service time is to be constant for all classes. Khamisy and Sidi [8] considered discrete-time priority queueing systems with two-state Markov modulated arrivals, where the service time is one slot for all classes. Gupta and Georganas [12] analyzed the input queueing switch. They modeled each input queue as a two-class Geo/G/1 queue with nonpreemptive priority and obtained the mean delay of two priority classes, respectively. Schorams [13] derived iterative algorithms that permit the evaluation of the steady state waiting and system time probabilities for two different priority classes in a discrete-time queueing system.

This paper considers a discrete-time $\mathrm{Geo}^X/\mathrm{G}/1$ queue accepting two classes of messages with preemptive resume priority. Bruneel and Kim [14], and Hashida [15] also studied the priority queue, but they only just obtained the marginal probability generating function (PGF) for the queue size of each priority classes. Using the generating function technique and the supplementary variable method, we derive the joint PGFs of the system occupancy distributions at various observation instant and provide the analysis of the system time and the busy period. Note that the supplementary variable method has successfully been applied to various queueing systems.

2. MODEL

There are two priority classes of messages; class-1 and class-2. A higher priority is assigned to class-1. Messages in each priority class arrive to the system in accordance with corresponding batch geometric process [6,16]. Let $A(z)$ (respectively, $B(z)$) be the PGF of the number a (respectively, b) of class-1 (respectively, class-2) messages that arrive in each slot. The number of messages of each class that arrive in the same slot is independent of each other. The system consists of two separate buffers of infinite capacity to accommodate arriving messages of corresponding priority classes.

There is a single server. Messages are served under the preemptive resume priority discipline. Thus, messages of high priority interrupt the service of a message of low priority and the message of low priority on its re-entry resumes its service from the preempted point. Messages in the same class are served in FIFO order. The service times of messages in each priority class are independent and identically distributed in accordance with a general probability distribution which may differ between two priority classes. Let $S_i(z)$ denote the PGF of the service time s_i for class-i, $i = 1, 2$. All messages arriving to the system are assumed to be eventually served, i.e., $A'(1)S_1'(1) + B'(1)S_2'(1) < 1$. The service times and the arrival processes are also assumed to be mutually independent.

Before proceeding to the analysis, we define some random variables. Let a random variable $n_k^{(i)}$ indicate the number of class-i messages in the queue (excluding a possible one in service or in limbo) at the end of slot k. A supplementary random variable $h_k^{(i)}$ is defined as follows: $h_k^{(i)}$ indicates the remaining service time if a class-i message is in service or in limbo at the end of slot k, and otherwise $h_k^{(i)} = 0$. Then $\{(h_k^{(1)}, n_k^{(1)}, h_k^{(2)}, n_k^{(2)})\}$ constitutes a Markov chain embedded at the end of each slot.

3. EMBEDDED MARKOV CHAIN

If we denote by a_k and b_k the numbers of class-1 and class-2 messages, respectively, entering the system during slot k, then the system under consideration evolves as follows.

(a) If $h_k^{(1)} = h_k^{(2)} = 0$:

$$n_{k+1}^{(1)} = \left(n_k^{(1)} - 1\right)^+ + a_{k+1};$$

$$n_{k+1}^{(2)} = \begin{cases} \left(n_k^{(2)} - 1\right)^+ + b_{k+1}, & \text{if } n_k^{(1)} = 0, \\ n_k^{(2)} + b_{k+1}, & \text{if } n_k^{(1)} > 0; \end{cases}$$
$$h_{k+1}^{(1)} = \begin{cases} 0, & \text{if } n_k^{(1)} = 0, \\ s_1 - 1, & \text{if } n_k^{(1)} > 0; \end{cases}$$
$$h_{k+1}^{(2)} = \begin{cases} 0, & \text{if } n_k^{(1)} > 0 \text{ or } n_k^{(2)} = 0, \\ s_2 - 1, & \text{if } n_k^{(1)} = 0 \text{ and } n_k^{(2)} > 0. \end{cases}$$

(b) If $h_k^{(1)} = 0$ and $h_k^{(2)} > 0$:

$$n_{k+1}^{(1)} = \left(n_k^{(1)} - 1\right)^+ + a_{k+1};$$
$$n_{k+1}^{(2)} = n_k^{(2)} + b_{k+1};$$
$$h_{k+1}^{(1)} = \begin{cases} 0, & \text{if } n_k^{(1)} = 0, \\ s_1 - 1, & \text{if } n_k^{(1)} > 0; \end{cases}$$
$$h_{k+1}^{(2)} = \begin{cases} h_k^{(2)} - 1, & \text{if } n_k^{(1)} = 0, \\ h_k^{(2)}, & \text{if } n_k^{(1)} > 0. \end{cases}$$

(c) If $h_k^{(1)} > 0$:

$$n_{k+1}^{(1)} = n_k^{(1)} + a_{k+1};$$
$$n_{k+1}^{(2)} = n_k^{(2)} + b_{k+1};$$
$$h_{k+1}^{(1)} = h_k^{(1)} - 1;$$
$$h_{k+1}^{(2)} = h_k^{(2)}.$$

Now, let us define $P_k(x, z, y, w)$ as the joint PGF of the state vector $(h_k^{(1)}, n_k^{(1)}, h_k^{(2)}, n_k^{(2)})$, valid at the end of slot k

$$P_k(x, z, y, w) \equiv E\left[x^{h_k^{(1)}} z^{n_k^{(1)}} y^{h_k^{(2)}} w^{n_k^{(2)}}\right]. \tag{1}$$

The next step is to derive a relationship between $P_k(x, z, y, w)$ and $P_{k+1}(x, z, y, w)$ by using the above state equations. We proceed as follows:

$$\begin{aligned} P_{k+1}(x, z, y, w) = A(z)B(w)\bigg[\frac{1}{x} P_k(x, z, y, w) + \left\{\frac{S_1(x)}{xz} - \frac{1}{x}\right\} P_k(0, z, y, w) \\ + \left\{\frac{1}{y} - \frac{S_1(x)}{xz}\right\} P_k(0, 0, y, w) + \left\{\frac{S_2(y)}{yw} - \frac{1}{y}\right\} P_k(0, 0, 0, w) + \left\{1 - \frac{S_2(y)}{yw}\right\} P_k(0, 0, 0, 0)\bigg]. \end{aligned} \tag{2}$$

A steady-state joint PGF is defined as

$$P(x, z, y, w) \equiv \lim_{k \to \infty} P_k(x, z, y, w), \tag{3}$$

provided that the system reaches a steady state. Letting $k \to \infty$ in equation (2) and solving for $P(x, z, y, w)$, we get

$$\begin{aligned} [x - A(z)B(w)]P(x, z, y, w) = xA(z)B(w)\bigg[\left\{\frac{S_1(x)}{xz} - \frac{1}{x}\right\} P(0, z, y, w) \\ + \left\{\frac{1}{y} - \frac{S_1(x)}{xz}\right\} P(0, 0, y, w) + \left\{\frac{S_2(y)}{yw} - \frac{1}{y}\right\} P(0, 0, 0, w) + \left\{1 - \frac{S_2(y)}{yw}\right\} P(0, 0, 0, 0)\bigg]. \end{aligned} \tag{4}$$

Since $|A(z)B(w)| \leq 1$ and $P(x,z,y,w)$ is analytical function in the closed polydisk $|x|$, $|y|$, $|z|$, $|w| \leq 1$, the right-hand side (RHS) of equation (4) must be zero for $x = A(z)B(w)$. Therefore, choosing $x = A(z)B(w)$ in equation (4), we have

$$
\begin{aligned}
[z - S_1(A(z)B(w))]P(0,z,y,w) &= \left[\frac{zA(z)B(w)}{y} - S_1(A(z)B(w))\right] P(0,0,y,w) \\
&+ \frac{zA(z)B(w)}{y}\left[\frac{S_2(y)}{w} - 1\right] P(0,0,0,w) + zA(z)B(w)\left[1 - \frac{S_2(y)}{yw}\right] P(0,0,0,0).
\end{aligned} \tag{5}
$$

We will show that, for a given w with $|w| \leq 1$, $z - S_1(A(z)B(w)) = 0$ has a unique root $z = z^*(w)$ in $|z| \leq 1$. We define $f(z) \equiv z$ and $g(z) \equiv -S_1(A(z)B(w))$. Substituting $z = z_0 + \triangle z$ for $|z_0| = 1$, $|\triangle z| \ll 1$ and $|w| \leq 1$, we have $g(z_0 + \triangle z) = g(z_0) + \triangle z[\frac{dg(z)}{dz}]_{z=z_0} + o(\triangle z)$ and

$$
|g(z_0 + \triangle z)| \leq |g(z_0)| + \left|\triangle z \left[\frac{dg(z)}{dz}\right]_{z=z_0}\right| + o(\triangle z). \tag{6}
$$

Since $|g(z_0)| = |S_1(A(z_0)B(w))| \leq 1$ and $|[\frac{dg(z)}{dz}]_{z=z_0}| \leq A'(1)S_1'(1)$, we get $|g(z_0 + \triangle z)| \leq 1 + A'(1)S_1'(1)|\triangle z| + o(\triangle z)$. Hence, on $|z| = 1 + \epsilon$ for a small positive real ϵ, $|g(z)| \leq 1 + A'(1)S_1'(1)\epsilon + o(\epsilon)$ and obviously $|f(z)| = 1 + \epsilon$. Since $|f(z)| > |g(z)|$ on $|z| = 1 + \epsilon$, by Rouché's theorem [16], $f(z)$ and $f(z) + g(z)$ have the same number of zeros inside $|z| = 1 + \epsilon$. Since $f(z)$ has a unique zero inside $|z| = 1 + \epsilon$, $z - S_1(A(z)B(w)) = 0$ has a unique root $z = z^*(w)$ in the closed disk $|z| \leq 1$. Since $P(0,z,y,w)$ is analytical function for $|z| \leq 1$, the RHS of (5) must be zero for $z = z^*(w)$. Therefore, we have

$$
\begin{aligned}
&[y - A(z^*(w))B(w)]P(0,0,y,w) \\
&= A(z^*(w))B(w)\left[\left\{\frac{S_2(y)}{w} - 1\right\} P(0,0,0,w) + \left\{y - \frac{S_2(y)}{w}\right\} P(0,0,0,0)\right].
\end{aligned} \tag{7}
$$

If we choose $y = A(z^*(w))B(w)$ in equation (7), the RHS of equation (7) must then vanish for $|y|, |w| \leq 1$, which yields

$$
P(0,0,0,w) = \frac{wA(z^*(w))B(w) - S_2(A(z^*(w))B(w))}{w - S_2(A(z^*(w))B(w))} P(0,0,0,0). \tag{8}
$$

By taking the derivative with respect to x of equation (4) and putting $x = z = y = w = 1$, we get

$$
P(0,0,0,0) = 1 - A'(1)S_1'(1) - B'(1)S_2'(1). \tag{9}
$$

Hence, the PGF $P(x,z,y,w)$ is determined.

4. SYSTEM OCCUPANCY

In this section, we will derive expression for the PGFs of the system occupancy measured at four different sets of time instant. More specifically, we will study the following steady state random variables:

$n^{(i)} \equiv$ queue occupancy of class-i messages at random slot boundaries,
$r^{(i)} \equiv$ system occupancy of class-i messages at random slot boundaries,
$d_j^{(i)} \equiv$ system occupancy of class-i messages at departure times of class-j messages,
$d^{(i)} \equiv$ system occupancy of class-i messages at departure times of any message,
$c^{(i)} \equiv$ system occupancy of class-i messages at arrival times of class-i messages,
$q^{(i)} \equiv$ system occupancy of class-i messages at random time points.

The corresponding PGFs are indicated as

$$N(z,w) \equiv E\left[z^{n^{(1)}} w^{n^{(2)}}\right], \qquad R(z,w) \equiv E\left[z^{r^{(1)}} w^{r^{(2)}}\right], \tag{10}$$

$$D_j(z,w) \equiv E\left[z^{d_j^{(1)}} w^{d_j^{(2)}}\right], \qquad D(z,w) \equiv E\left[z^{d^{(1)}} w^{d^{(2)}}\right], \tag{11}$$

$$C^{(i)}(z) \equiv E\left[z^{c^{(i)}}\right], \qquad Q(z,w) \equiv E\left[z^{q^{(1)}} w^{q^{(2)}}\right], \tag{12}$$

respectively.

A. System Occupancy at Random Slot Boundaries

According to our developments in the previous section, $N(z,w)$ can be obtained for $P(x,z,y,w)$ by simply putting $x = y = 1$

$$N(z,w) = P(1,z,1,w). \tag{13}$$

Since

$$r^{(i)} = n^{(i)} + \min\left(h^{(i)}, 1\right), \tag{14}$$

for $i = 1, 2$, the joint PGF $R(z,w)$ is given by

$$R(z,w) = (1-z)(1-w)P(0,z,0,w) + w(1-z)P(0,z,1,w) + z(1-w)P(1,z,1,w) + zwP(1,z,1,w).$$

B. System Occupancy at Departure Times

The random variable $d_j^{(i)}$ can be expressed as

$$d_1^{(1)} = u_1^{(1)} + a; \qquad d_1^{(2)} = u_1^{(2)} + b; \qquad d_2^{(1)} = a; \qquad d_2^{(2)} = u_2^{(2)} + b, \tag{15}$$

where $u_1^{(i)}$ denotes the number of class-i messages in the queue at the end of an arbitrary slot with $h^{(1)} = 1$, and $u_2^{(2)}$ denotes the number of class-2 messages in the queue at the end of an arbitrary slot with $h^{(2)} = 1$ and $n^{(1)} = 0$. Therefore, we obtain

$$\begin{aligned} D_1(z,w) &= \frac{1 - S_1'(1)A'(1) - S_2'(1)B'(1)}{A'(1)\left[1 - S_1'(0)\right]} \times \frac{w-1}{w} \times \frac{wA\left(z^*(w)\right)B(w) - S_2\left(A\left(z^*(w)\right)B(w)\right)}{w - S_2\left(A\left(z^*(w)\right)B(w)\right)} \\ &\quad \times \left[1 - \frac{zA(z)B(w)}{z - S_1(A(z)B(w))} - \frac{A\left(z^*(w)\right)B(w)}{1 - A\left(z^*(w)\right)B(w)} \left\{\frac{S_1'(0)}{z} - \frac{z(1-A(z)B(w))}{z - S_1(A(z)B(w))}\right\}\right], \\ D_2(z,w) &= \frac{1}{B'(1)\left[1 - S_2'(0)\right]} \left[\left\{A\left(z^*(w)\right)B(w) - \frac{S_2'(0)}{w}\right\} P(0,0,0,w) \right. \\ &\quad \left. + \left\{\frac{S_2'(0)}{w} - 1\right\} P(0,0,0,0)\right], \end{aligned}$$

and

$$D(z,w) = A(z)B(w) \frac{P\left\{h^{(1)} = 1\right\} D_1(z,w) + P\left\{h^{(2)} = 1, n^{(1)} = 0\right\} D_2(z,w)}{P\left\{h^{(1)} = 1\right\} + P\left\{h^{(2)} = 1, n^{(1)} = 0\right\}}. \tag{16}$$

C. System Occupancy as Seen by New Arrivals

To investigate the probability distribution of the random variable $c^{(i)}$, we need more detailed information on the way in which messages enter the queueing system. First, the numbers of arrival instants falling in the consecutive slots for each class are assumed to be independent and identically distributed. Let t_i be the number of arrival instants of class-i in one slot and $T_i(z)$ the PGF of t_i. Next, the numbers of messages entering the system at each arrival instant are also assumed to be independent and identically distributed. Let l_i be the bulk size of class-i

messages at each arrival instant and $L_i(z)$ be the PGF of l_i. Then, the PGFs $A(z)$ and $B(z)$ can be obtained from

$$A(z) = T_1(L_1(z)), \qquad B(z) = T_2(L_2(z)). \tag{17}$$

Consider a tagged message that arrives at the system. Let us refer to the slot in which its arrival instant is situated as the tagged slot. Now let the discrete random variable g_i be the number of class-i messages entering the system during the tagged slot before the arrival instant of the tagged message. Then, the PGF $G_i(z)$ of g_i is given by

$$G_1(z) = \frac{A(z) - 1}{[L_1(z) - 1]T_1'(1)}, \qquad G_2(z) = \frac{B(z) - 1}{[L_2(z) - 1]T_2'(z)}. \tag{18}$$

Since

$$c^{(i)} = r^{(i)} + g_i, \tag{19}$$

it follows that

$$C^{(1)}(z) = \frac{A(z) - 1}{[L_1(z) - 1]T_1'(1)} R(z, 1), \qquad C^{(2)}(z) = \frac{B(z) - 1}{[L_2(z) - 1]T_2'(1)} R(1, z). \tag{20}$$

D. System Occupancy at Random Time Points

To investigate the probability distribution of the random variable $q^{(i)}$, we must have a more detailed description of the exact location of each arrival instant within a slot. The position of each arrival instant within the slot is assumed to be i.i.d. continuous random variable. The position of an arrival instant within a slot is characterized by specifying its distance p_i from the beginning of the slot. Let $P_i(\theta)$ be the corresponding cumulative distribution function. Consider an arbitrary continuous time instant t and refer to the slot to which t belongs as tagged slot. Let the random variable m_i be the number of class-i messages entering the system during the tagged slot before t. Then, the PGF $M_i(z)$ of m_i is given by

$$M_i(z) = \int_0^1 T_i(1 - P_i(\theta) + P_i(\theta)L_i(z))\, d\theta, \tag{21}$$

for $i = 1, 2$. Since the system occupancy observed at random time points can now be obtained as

$$q^{(i)} = r^{(i)} + m_i, \tag{22}$$

it thus, follows that:

$$Q(z, w) = R(z, w)M_1(z)M_2(w). \tag{23}$$

5. UNFINISHED WORK AND SYSTEM TIME

Let $w_k^{(i)}$ be the unfinished work of class-i at the end of slot k. Then, $w_k^{(i)}$ is given by

$$w_k^{(i)} = h_k^{(i)} + \sum_{j=1}^{n_k^{(i)}} s_i^{(j)}, \tag{24}$$

for $i = 1, 2$, where $s_i^{(j)}$, $j = 1, 2, \dots, n_k^{(i)}$, are mutually independent with the same distribution as s_i. Hence, the joint PGF $W(z, w)$ of the unfinished works is

$$W(z, w) = P(z, S_1(z), w, S_2(w)). \tag{25}$$

Now, the system time of the message is studied. Let us refer to the arrival slot of a tagged message as a tagged slot and let v_i denote the system time of the tagged class-i message. Then, the random variable v_i can be expressed as follows:

$$v_1 = \left(w^{(1)} - 1\right)^+ + \sum_{i=1}^{f_1} s_1^{(i)} + s_1, \tag{26}$$

$$v_2 = e\left(\left(w^{(1)} + w^{(2)} - 1\right)^+ + \sum_{i=1}^{a} s_1^{(i)} + \sum_{i=1}^{f_2} s_2^{(i)} + s_2 - 1\right) + 1, \tag{27}$$

where f_i denotes the number of messages that will be served before the tagged message among those messages arriving during the tagged slot and $e(n)$ denotes the busy period of class-1 generated by n slots. The PGF $V_i(z)$ of v_i is given by

$$V_1(z) = \frac{P(z, S_1(z), 1, 1) + (z-1)P(0,0,1,1)}{z} F_1(S_1(z))S_1(z), \tag{28}$$

$$\begin{aligned} V_2(z) = {} & \frac{P(E(z), S_1(E(z)), E(z), S_2(E(z))) + (E(z)-1)P(0,0,0,0)}{E(z)} \\ & \times F_2(S_2(E(z)))A(S_1(E(z)))\frac{S_2(z)}{E(z)}z, \end{aligned} \tag{29}$$

where the PGF $F_i(z)$ of f_i is given by

$$F_1(z) = \frac{A(z) - 1}{(z-1)A'(1)}, \qquad F_2(z) = \frac{B(z) - 1}{(z-1)B'(z)}, \tag{30}$$

and the busy period $e(1)$ is the same as that of Geo^X/G/1 queue without priority and the implicit formula for its PGF $E(z)$ is given in [14].

6. IDLE AND BUSY PERIOD

Let i^* be the length of an arbitrary idle period and $I^*(z)$ the PGF of i^*. Since an idle period will last for k consecutive slots if and only if there are no arrivals during each of the first $k-1$ of these slots and at least one arrival during the k^{th} of these slots,

$$P[i^* = k] = [1 - A(0)B(0)][A(0)B(0)]^{k-1}, \qquad k \geq 1,$$
$$I^*(z) = \frac{[1 - A(0)B(0)]z}{1 - A(0)B(0)z}.$$

Let e^* be the length of an arbitrary busy period and $E^*(z)$ the PGF of e^*. After an idle period, a message enters service introducing a new busy period. Let e_i indicate the length of the time period during which the server is occupied by a class-i message and its successors, and $E_i(z)$ the PGF of e_i. The whole busy period can be partitioned in $a^* + b^*$ consecutive sub-busy periods, where a^* and b^* denote the number of arrivals of class-1 and class-2, respectively, during the last slot of an idle period. Thus,

$$e^* = \sum_{j=1}^{a^*} e_1^{(j)} + \sum_{j=1}^{b^*} e_2^{(j)}, \tag{33}$$

where $e_i^{(j)}$ are mutually independent with the same distribution as e_i for each i. Since the joint PGF of a^* and b^* is

$$\frac{A(z)B(w) - A(0)B(0)}{1 - A(0)B(0)},$$

the PGF $E^*(z)$ is given by

$$E^*(z) = \frac{A(E_1(z))B(E_2(z)) - A(0)B(0)}{1 - A(0)B(0)}. \tag{34}$$

Since the sub-busy period e_i starts with the service time s_i, the period e_i can be expressed as

$$e_i = s_i + \sum_{j=1}^{S_i^{(1)}} e_1^{(j)} + \sum_{j=1}^{S_i^{(2)}} e_2^{(j)}, \tag{35}$$

where $S_i^{(j)}$ denotes the number of class-j messages entering the system during s_i. Thus,

$$E_i(z) = S_i(zA(E_1(z))B(E_2(z))), \tag{36}$$

for $i = 1, 2$.

7. CONCLUDING REMARKS

This paper considered a $\mathrm{Geo}^X/\mathrm{G}/1$ queue with preemptive resume priority. At various observation instant, an analysis of the joint system occupancy distributions was provided by means of probability generating functions and supplementary variable method. Further, we obtained implicit formulas for the probability distribution of the system time and the busy period.

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