Priority queueing system with fixed-length packet-train arrivals

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Abstract: A discrete-time priority queueing system is studied, in which two different classes of fixed-length packet-trains arrive according to independent batch geometric streams. The packets in each packet-train arrive at the rate of one packet per slot (train arrivals), resulting in a correlated arrival stream. The service time of one packet is deterministic of one slot. The motivation for the work comes from ATM networks with diverse traffic sources and correlated packet arrival stream. Using the probability generating function method, the joint distribution of queue lengths and the waiting time distribution are obtained for each class. Numerical results are presented. Comparision is made with the case of 'batch arrivals', where all packets of a packet-train arrive simultaneously at the buffer.

1 Introduction

We analyse a discrete-time priority queueing system with fixed-length packet-train arrivals. As in usual discrete-time systems [1–4], the time axis is segmented into equal intervals of unit duration called slots. The length of one slot is the time interval required to transmit exactly one packet. It is assumed that arrivals and services occur at slot boundaries; arrival and service completion of a packet occur just before slot boundaries. The packets arrive by fixed-length packet-train of size m [5]; if the leading packet of a message containing m packets arrives in the current slot, the remaining $m - 1$ packets of that message will arrive in the next $m - 1$ slots.

In ATM networks [3], all information is transmitted by fixed-size packet of 53 bytes. Thus, ATM networks can usually be analysed by a discrete-time model. The discrete-time queue has been investigated fairly extensively. The presence of packet-train arrivals in input streams of ATM networks comes from the segmentation of large data messages into small cells. As an application for an ATM multipath self-routing switch [6], Xiong and Bruneel [5] considered a discrete-time single-server queue with a 'deterministically correlated' arrival process, called a fixed-length packet-train arrival process. The messages are concretely segmented into packets, and the number of packets in a message is assumed to be constant. Thus, each message looks like a fixed-length packet-train which enters the buffer at the rate of one packet per slot. Xiong and Bruneel obtained [5] an explicit expression for the probability generating function of the queue length. Wittevrongel and Bruneel [7] extended this model to a variable-length packet-train model.

ATM networks support diverse services such as voice, data and video, which require different QoS requirements. For example, data is loss-sensitive but delay-insensitive, whereas voice is delay-sensitive but loss-insensitive. In ATM networks, one of the most important problems is to meet the QoS for all traffic, e.g. the delay and loss requirements for real-time and non-real-time traffic. One method of solving this problem is the use of priority control [1, 3, 4]. In this paper, we consider two classes of message (high and low priority) with fixed-length packet-train arrivals. High priority packets may be considered as real-time traffic such as voice, and low priority packets may be considered as a non-real-time traffic such as data. We extend the model of Xiong and Bruneel [5] with only one class to the model with two HOL priority classes.

Using the generating function method, we analyse the discrete-time priority queue with fixed-length packet-train arrivals and obtain the joint queue length distribution of high and low priority. The waiting time distribution for a packet in each class is derived explicitly in closed form. Since a message can be regarded as a fixed-length packet-train, the waiting time distribution for a message in the case of 'FCFS for messages in the same class' is also given.

2 Analytical model description

We consider a discrete-time single server priority queue with two classes of high priority and low priority. Each class arrives by packet-trains with a fixed size of m packets; if a leading packet of a message containing m

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packets arrives in the current slot, the remaining $m-1$ packets arrive consecutively in the next $m-1$ slots. The packets are accommodated in the buffers with infinite capacity. The service time of a packet is one slot. We assume that the numbers of arrivals of leading packets of high priority and low priority in a slot are independent and identically distributed with respective probability generating functions $A(z)$ and $B(z)$. We refer to high and low priority as type-1 and type-2, respectively.

Let the random variable $a_{k,n}$ (resp. $b_{k,n}$), $1 \le k \le m$ be the number of packet-trains of type-1 (resp. type-2) that generate the kth packet of a packet-train during slot n. It is obvious that the following relations hold for $2 \le k \le m$:

$$a_{k,n+1} = a_{k-1,n}, \quad b_{k,n+1} = b_{k-1,n} \tag{1}$$

Let a_{n+1} and b_{n+1} be the total number of packets of type-1 and type-2 entering the corresponding buffer in slot $n+1$, respectively. a_{n+1} and b_{n+1} can then be expressed as

$$a_{n+1} = \sum_{k=1}^{m} a_{k,n+1} = a_{1,n+1} + \sum_{k=2}^{m} a_{k-1,n} = \sum_{i=0}^{m-1} a_{1,n+1-i} \tag{2}$$

$$b_{n+1} = \sum_{k=1}^{m} b_{k,n+1} = b_{1,n+1} + \sum_{k=2}^{m} b_{k-1,n} = \sum_{i=0}^{m-1} b_{1,n+1-i} \tag{3}$$

where $a_{1,n+1-i}$ and $b_{1,n+1-i}$, $i = 0, 1, \ldots, m-1$, are independent and identically distributed with the common probability generating function $A(z)$ and $B(z)$, respectively. From eqns. 2 and 3, the offered load ρ_1 and ρ_2 for each type in a slot is given by $\rho_1 \equiv mA'(1)$ and $\rho_2 \equiv mB'(1)$. Let the random variable $N_1(n)$ (resp. $N_2(n)$) be the number of packets of type-1 (resp. type-2) accumulated in the corresponding buffer just after slot n. By considering the evolution of the system in two successive slots, we then have a set of equations:

$$N_1(n+1) = (N_1(n)-1)^+ + a_{n+1} \tag{4}$$

$$N_2(n+1) = \begin{cases} (N_2(n)-1)^+ + b_{n+1}, & \text{if } N_1(n) = 0 \\ N_2(n) + b_{n+1}, & \text{otherwise} \end{cases} \tag{5}$$

where $(x)^+ \equiv \max(x, 0)$

It is easy to see that the process

$$\{(a_{1,n}, \ldots, a_{m-1,n}, N_1(n), b_{1,n}, \ldots, b_{m-1,n}, N_2(n)),\ n \ge 0\} \tag{6}$$

is a $2m$-dimensional Markov chain. Below we derive the joint queue length distribution in the steady-state.

3 Joint queue length distribution

We introduce the $2m$-dimensional joint probability generating function of the random variables $a_{1,n}, \ldots, a_{m-1,n}$, $N_1(n)$, $b_{1,n}, \ldots, b_{m-1,n}$ and $N_2(n)$:

$$R_n(x_1, \ldots, x_{m-1}, z_1, y_1, \ldots, y_{m-1}, z_2) \equiv E\left[x_1^{a_{1,n}} \cdots x_{m-1}^{a_{m-1,n}} z_1^{N_1(n)} y_1^{b_{1,n}} \cdots y_{m-1}^{b_{m-1,n}} z_2^{N_2(n)}\right] \tag{7}$$

From eqns. 1–5, it follows that

$$\begin{aligned} &R_{n+1}(x_1, \ldots, x_{m-1}, z_1, y_1, \ldots, y_{m-1}, z_2) \\ &= E\left[x_1^{a_{1,n+1}} \cdots x_{m-1}^{a_{m-1,n+1}} z_1^{N_1(n+1)} y_1^{b_{1,n+1}} \cdots \cdots y_{m-1}^{b_{m-1,n+1}} z_2^{N_2(n+1)}\right] \\ &= A(x_1 z_1) B(y_1 z_2) \left[\frac{1}{z_1} R_n(x_2 z_1, \ldots, x_{m-1} z_1, z_1, z_1, y_2 z_2, \ldots, y_{m-1} z_2, z_2, z_2) \right. \\ &\quad + \left(\frac{1}{z_2} - \frac{1}{z_1}\right) E\left[(y_2 z_2)^{b_{1,n}} \cdots \cdots (y_{m-1} z_2)^{b_{m-2,n}} z_2^{b_{m-1,n}} z_2^{N_2(n)} 1_{\{N_1(n)=0\}}\right] \\ &\quad \left. + \left(1 - \frac{1}{z_2}\right) P\{N_1(n) = 0, N_2(n) = 0\}\right] \end{aligned} \tag{8}$$

Note that $P\{N_1(n) = 0, N_2(n) = 0\}$ is the probability that both buffers are empty just after slot n. Since $N_1(n) = 0$ implies that $a_{k,n} = 0$, $1 \le k \le m$, eqn. 8 becomes

$$\begin{aligned} &R_{n+1}(x_1, \ldots, x_{m-1}, z_1, y_1, \ldots, y_{m-1}, z_2) \\ &= A(x_1 z_1) B(y_1 z_2) \left[\frac{1}{z_1} R_n(x_2 z_1, \ldots, x_{m-1} z_1, z_1, z_1, y_2 z_2, \ldots, y_{m-1} z_2, z_2, z_2) \right. \\ &\quad + \left(\frac{1}{z_2} - \frac{1}{z_1}\right) R_n(0, \ldots, 0, y_2 z_2, \ldots, y_{m-1} z_2, z_2, z_2) \\ &\quad \left. + \left(1 - \frac{1}{z_2}\right) P\{N_1(n) = 0, N_2(n) = 0\}\right] \end{aligned} \tag{9}$$

We assume that $\rho_1 + \rho_2 < 1$ so as to reach the steady-state. Letting $n \to \infty$ in eqn. 9, we have

$$\begin{aligned} &R(x_1, \ldots, x_{m-1}, z_1, y_1, \ldots, y_{m-1}, z_2) \\ &= \lim_{n\to\infty} R_n(x_1, \ldots, x_{m-1}, z_1, y_1, \ldots, y_{m-1}, z_2) \\ &= A(x_1 z_1) B(y_1 z_2) \left[\frac{1}{z_1} R(x_2 z_1, \ldots, x_{m-1} z_1, z_1, z_1, y_2 z_2, \ldots, y_{m-1} z_2, z_2, z_2) \right. \\ &\quad + \left(\frac{1}{z_2} - \frac{1}{z_1}\right) R(0, \ldots, 0, y_2 z_2, \ldots, y_{m-1} z_2, z_2, z_2) \\ &\quad \left. + \left(1 - \frac{1}{z_2}\right) p_0\right] \end{aligned} \tag{10}$$

where p_0 denotes the probability that both buffers are empty at an arbitrary slot in the steady state.

Let $N(z_1, z_2)$ be the joint probability generating function of queue lengths in the steady-state. Setting $x_1 = \ldots = x_{m-1} = y_1 = \ldots = y_{m-1} = 1$ in eqn. 10 we then obtain

$$\begin{aligned} N(z_1, z_2) &= R(1, \ldots, 1, z_1, 1, \ldots, 1, z_2) \\ &= A(z_1) B(z_2) \left[\frac{1}{z_1} R(z_1, \ldots, z_1, z_2, \ldots, z_2) \right. \\ &\quad + \left(\frac{1}{z_2} - \frac{1}{z_1}\right) R(0, \ldots, 0, z_2, \ldots, z_2) \\ &\quad \left. + \left(1 - \frac{1}{z_2}\right) p_0\right] \end{aligned} \tag{11}$$

Next, we find out $R(z_1, ..., z_1, z_2, ..., z_2)$ and p_0. Define the probability generating function $T_k(z_1, z_2)$ as

$$T_k(z_1,z_2) \equiv R(x_1,\dots,x_{m-1},z_1,y_1,\dots,y_{m-1},z_2)$$
$$1 \le k \le m-1$$

where

$$\begin{aligned} x_1 = \cdots = x_{m-k} &= z_1^k \\ x_{m-k+1} = z_1^{k-1}, \cdots, x_{m-1} &= z_1 \\ y_1 = \cdots = y_{m-k} &= z_2^k \\ y_{m-k+1} = z_2^{k-1}, \cdots, y_{m-1} &= z_2 \end{aligned} \tag{12}$$

Then $T_1(z_1, z_2) = R(z_1, ..., z_1, z_2, ..., z_2)$ and $T_1(0, z_2) = R(0, ..., 0, z_2, ..., z_2)$. From eqn. 10, $T_k(z_1, z_2)$ can be written in the following recursive form:

$$\begin{aligned} T_k(z_1,z_2) = A(z_1^{k+1})B(z_2^{k+1}) \bigg[&\frac{1}{z_1} T_{k+1}(z_1,z_2) \\ &+ \left(\frac{1}{z_2} - \frac{1}{z_1}\right) T_{k+1}(0,z_2) + \left(1 - \frac{1}{z_2}\right) p_0 \bigg], \\ &1 \le k \le m-2 \end{aligned} \tag{13}$$

$$\begin{aligned} T_{m-1}(z_1,z_2) = A(z_1^m)B(z_2^m) \bigg[&\frac{1}{z_1} T_{m-1}(z_1,z_2) \\ &+ \left(\frac{1}{z_2} - \frac{1}{z_1}\right) T_{m-1}(0,z_2) + \left(1 - \frac{1}{z_2}\right) p_0 \bigg] \end{aligned} \tag{14}$$

If we solve eqn. 14 for $T_{m-1}(z_1, z_2)$, we then have

$$T_{m-1}(z_1,z_2) = \frac{A(z_1^m)B(z_2^m)[z_1(z_2-1)p_0 + (z_1 - z_2)T_{m-1}(0,z_2)]}{z_2[z_1 - A(z_1^m)B(z_2^m)]} \tag{15}$$

By Rouche's theorem, there exists a unique solution $z_1 = z_0(z_2)$ $(= z_0)$ of $z_1 = A(z_1^m)B(z_2^m)$ in $|z_1| < 1$ for each z_2 with $|z_2| < 1$. The solution $z_0(z_2)$ is expressed as

$$z_0(z_2)(= z_0) = \sum_{n=1}^{\infty} \frac{B^n(z_2^m)}{n!} \frac{d^{n-1}}{dz_1^{n-1}} A^n(z_1^m) \bigg|_{z_1=0} \tag{16}$$

$T_{m-1}(0, z_2)$ is then given by

$$T_{m-1}(0,z_2) = \frac{z_0(z_2-1)p_0}{z_2 - z_0} \tag{17}$$

From eqns. 14 and 17, we have

$$\begin{aligned} T_{m-1}&(z_1,z_2) \\ &= \frac{A(z_1^m)B(z_2^m)[z_1(z_2-1)p_0 + (z_1-z_2)\frac{z_0(z_2-1)}{z_2-z_0}p_0]}{z_2[z_1 - A(z_1^m)B(z_2^m)]} \\ &= \frac{A(z_1^m)B(z_2^m)(z_2-1)p_0\left[z_1 + \frac{z_0(z_1-z_2)}{z_2-z_0}\right]}{z_2[z_1 - A(z_1^m)B(z_2^m)]} \\ &= \frac{A(z_1^m)B(z_2^m) + \frac{z_0(z_1-z_0)(z_2-1)}{z_2-z_0}p_0}{z_2[z_1 - A(z_1^m)B(z_2^m)]} \\ &= \frac{(z_1-z_0)A(z_1^m)B(z_2^m)(z_2-1)p_0}{(z_2-z_0)[z_1 - A(z_1^m)B(z_2^m)]} \end{aligned} \tag{18}$$

Therefore, using the eqns. 13, 17, and 18, we can derive $T_1(z_1, z_2)$ and $T_1(0, z_2)$ recursively. Finally, the joint probability generating function $N(z_1, z_2)$ is given as

$$\begin{aligned} N(z_1,z_2) &= R(1,\dots,1,z_1,1,\dots,1,z_2) \\ &= A(z_1)B(z_2)\bigg[\frac{1}{z_1}R(z_1,\dots,z_1,z_2,\dots,z_2) \\ &\quad + \left(\frac{1}{z_2} - \frac{1}{z_1}\right) R(0,\dots,0,z_2,\dots,z_2) \\ &\quad + \left(1 - \frac{1}{z_2}\right) p_0\bigg] \\ &= A(z_1)B(z_2)\bigg[\frac{1}{z_1}T_1(z_1,z_2) \\ &\quad + \left(\frac{1}{z_2} - \frac{1}{z_1}\right) T_1(0,z_2) + \left(1 - \frac{1}{z_2}\right) p_0\bigg] \end{aligned} \tag{19}$$

The unknown quality p_0 remains to be obtained. By letting $z_1 = z_2 = z$ in eqn. 15, we obtain

$$T_{m-1}(z,z) = \frac{(z-1)A(z^m)B(z^m)}{z - A(z^m)B(z^m)} p_0 \tag{20}$$

By taking $z = 1$ in eqn. 20 and using L'Hòpital's rule, the unknown quality p_0 is given by $p_0 = 1 - \rho_1 - \rho_2$.

4 Waiting time analysis

We derive the waiting time distribution for a packet and a message of each type. We define the waiting time of a packet as the time period between the end of the packet's arrival slot and the time instant at which the transmission of this packet is about to start. The waiting time of a message (packet-train) is defined as the time period between the end of the slot during which the first packet of a message was generated and the time instant at which the transmission of this packet is about to start.

The waiting time for message is derived under the assumption that the service order of messages in the same class is FCFS and the messages arriving in the same slot are served randomly.

Since the packets of type-2, do not interfere with the waiting time of a type-1 packet the waiting time distribution of type-1 can be obtained as in the work by Xiong and Bruneel [5]. Hence, we focus on the waiting time analysis of type-2.

4.1 Waiting time distribution of a type-2 packet

Here we derive the waiting time W_2 of an arbitrary tagged type-2 packet. Let U be an initial delay consisting of the following time intervals:

(i) the unfinished work at arrival slot of a tagged type-2 packet; here the unfinished work is the time interval required to service all packets in the system, except for the packets arriving during the arrival slot of the tagged packet.

(ii) the service times of type-1 packets, which will 'be arrived' consecutively by the arrived leading packets until the arrival slot of the tagged packet.

(iii) the service times of packets, which will be served first among packets arriving during the arrival slot of the tagged packet.

To obtain the waiting time distribution of the tagged type-2 packet, we first need to calculate the probability distribution of this initial delay U. Let $q(i_1, \ldots, i_m, j_1, i, j_2)$ be the probability that there are j_1 type-1 packets and j_2 type-2 packets at the beginning of arrival slot of the tagged type-2 packet. There also are i_k kth packets of the type-1 packet-trains and i type-2 packets arriving in the slot where the tagged type-2 packet arrives. Let $Q(x_1, \ldots, x_m, z_1, y, z_2)$ be the corresponding joint probability generating function of $q(i_1, \ldots, i_m, j_1, i, j_2)$. The probability distribution of U is then given easily by

$$P\{U = k\} = \sum_{l=0}^{k} \sum_{i=k+1-l}^{\infty} \frac{q(i_1, \ldots, i_m, j_1, i, j_2)}{\rho_2} \tag{21}$$

where $l = (j_1 + j_2 - 1)^+ + mi_1 + (m-1)i_2 + \ldots + i_m$.

Thus, we can calculate the probability generating function $U(z)$ of the random variable U as follows:

$$\begin{aligned}
U(z) &= \sum_{k=0}^{\infty} P\{U = k\} z^k \\
&= \frac{1}{\rho_2} \sum_{k=0}^{\infty} \sum_{l=0}^{k} \sum_{i=k+1-l}^{\infty} q(i_1, \ldots, i_m, j_1, i, j_2) z^k \\
&= \frac{1}{\rho_2} \sum_{\substack{i_1,\ldots,i_m \\ j_1, j_2}} \sum_{u=0}^{\infty} \sum_{i=u+1}^{\infty} q(i_1, \ldots, i_m, j_1, i, j_2) z^{u+l} \\
&= \frac{1}{\rho_2(z-1)} \left[\frac{1}{z} \{ Q(z^m, \ldots, z, z, z, z) \right. \\
&\quad - Q(z^m, \ldots, z, z, 1, z)\} \\
&\quad + \left(1 - \frac{1}{z}\right) \{ Q(z^m, \ldots, z, 0, z, 0) \\
&\quad \left. - Q(z^m, \ldots, z, 0, 1, 0)\} \right] \\
&= \frac{1}{\rho_2(z-1)} \left[\frac{1}{z} \{ Q(z^m, \ldots, z, z, z, z) \right. \\
&\quad - Q(z^m, \ldots, z, z, 1, z)\} \\
&\quad \left. + \left(1 - \frac{1}{z}\right) \{ A(z^m)B(z)p_0 - A(z^m)p_0 \} \right]
\end{aligned} \tag{22}$$

Now $Q(x_1, x_2, \ldots, x_m, z_1, y, z_2)$ can be expressed by the probability generating function $R(x_1, \ldots, x_{m-1}, z_1, y_1, \ldots, y_{m-1}, z_2)$ as follows:

$$\begin{aligned}
Q(x_1, x_2, \ldots, x_m, z_1, y, z_2) \\
= A(x_1)B(y)R(x_2, \ldots, x_m, z_1, y, \ldots, y, z_2)
\end{aligned} \tag{23}$$

Therefore, eqns. 22 and 23 we obtain $U(z)$ as follows:

$$\begin{aligned}
U(z) = \frac{1}{\rho_2(z-1)} & \left[\frac{1}{z} A(z^m) \right. \\
& \times \{ B(z) R(z^{m-1}, \ldots, z, z, z, \ldots, z, z) \\
& - R(z^{m-1}, \ldots, z, z, 1, \ldots, 1, z) \} \\
& \left. + \left(1 - \frac{1}{z}\right) A(z^m) p_0 (B(z) - 1) \right]
\end{aligned} \tag{24}$$

From eqn. 10, it follows that

$$\begin{aligned}
& R(z^{m-1}, \ldots, z, 1, \ldots, 1, z) \\
&= A(z^m)B(z) \left\{ \frac{1}{z} R(z^{m-1}, \ldots, z, z, z, \ldots, z, z) \right. \\
&\qquad \left. + \left(1 - \frac{1}{z}\right) p_0 \right\} \\
&= \frac{A(z^m)B(z)}{z} R(z^{m-1}, \ldots, z, z, z, \ldots, z, z) \\
&\quad + A(z^m)B(z) \left(1 - \frac{1}{z}\right) p_0
\end{aligned} \tag{25}$$

By substituting eqn. 25 into eqn. 24, we obtain

$$\begin{aligned}
& U(z) \\
&= \frac{1}{\rho_2(z-1)} \left[\frac{1}{z} A(z^m) \left\{ B(z) R(z^{m-1}, \ldots, z, z, \ldots, z) \right. \right. \\
&\quad - \frac{A(z^m)B(z)}{z} R(z^{m-1}, \ldots, z, z, \ldots, z) \\
&\quad \left. - A(z^m)B(z) \left(1 - \frac{1}{z}\right) p_0 \right\} \\
&\quad \left. + \left(1 - \frac{1}{z}\right) A(z^m) p_0 (B(z) - 1) \right] \\
&= \frac{1}{\rho_2(z-1)} \left[\frac{1}{z} A(z^m) B(z) \left(1 - \frac{A(z^m)}{z}\right) \right. \\
&\quad \times R(z^{m-1}, \ldots, z, z, \ldots, z) \\
&\quad \left. + \left(1 - \frac{1}{z}\right) A(z^m) \left\{ B(z) - 1 - \frac{1}{z} A(z^m) B(z) \right\} p_0 \right]
\end{aligned} \tag{26}$$

Finally, $R(z^{m-1}, \ldots, z, z, z, \ldots, z, z)$ remains to be derived. As in Section 3, define the probability generating function $Z_k(z)$ as follows:

$$\begin{aligned}
Z_k(z) = R(z^{m-1}, \ldots, z, z, y_1, \ldots, y_{m-1}, z), \\
1 \le k \le m-1
\end{aligned} \tag{27}$$

where $y_1 = \ldots = y_{m-k} = z^k$, $y_{m-k+1} = z^{k-1}$, ..., $y_{m-1} = z$.

By eqn. 10, we then obtain a recursive form for $Z_k(z)$:

$$\begin{aligned}
Z_k(z) &= A(z^m)B(z^{k+1}) \left[\frac{1}{z} Z_{k+1}(z) + \left(1 - \frac{1}{z}\right) p_0 \right] \\
& \qquad 1 \le k \le m-2 \\
Z_{m-1}(z) &= A(z^m)B(z^m) \left[\frac{1}{z} Z_{m-1}(z) + \left(1 - \frac{1}{z}\right) p_0 \right]
\end{aligned} \tag{28}$$

From eqn. 28, $Z_{m-1}(z)$ is given by

$$Z_{m-1}(z) = \frac{A(z^m)B(z^m)(z-1)}{z - A(z^m)B(z^m)} p_0 \tag{29}$$

We also can derive $Z_1(z)$ recursively by eqns. 28 and 29:

$$\begin{aligned}
Z_1(z) &= \frac{A(z^m)B(z^2)}{z} Z_2(z) \\
&\quad + A(z^m)B(z^2) \left(1 - \frac{1}{z}\right) p_0 \\
&= \frac{A(z^m)B(z^2)}{z} \left[\frac{A(z^m)B(z^3)}{z} Z_3(z) \right.
\end{aligned}$$

$$+A(z^m)B(z^3)\left(1-\frac{1}{z}\right)p_0\Big]$$

$$+A(z^m)B(z^2)\left(1-\frac{1}{z}\right)p_0$$

$$=\frac{A(z^m)^{m-2}}{z^{m-2}}\prod_{l=2}^{m-1}B(z^l)Z_{m-1}(z)$$

$$+\left(1-\frac{1}{z}\right)p_0\sum_{h=1}^{m-2}\frac{A(z^m)^h}{z^{h-1}}\prod_{l=2}^{h+1}B(z^l)$$

$$=\frac{A(z^m)^{m-1}}{z^{m-2}}\frac{(z-1)}{z-A(z^m)B(z^m)}p_0\prod_{l=2}^{m}B(z^l)$$

$$+\left(1-\frac{1}{z}\right)p_0\sum_{h=1}^{m-2}\frac{A(z^m)^h}{z^{h-1}}\prod_{l=2}^{h+1}B(z^l) \tag{30}$$

By substituting $Z_1(z) = R(z^{m-1}, \ldots, z, z, z, \ldots, z, z)$ into eqn. 26, we finally obtain

$$U(z)=\frac{p_0A(z^m)}{\rho_2}\left[\frac{A(z^m)^{m-1}}{z^{m-1}}\left(1-\frac{A(z^m)}{z}\right)\right.$$

$$\times\frac{1}{z-A(z^m)B(z^m)}\prod_{l=1}^{m}B(z^l)$$

$$\left.+\frac{1}{z}\left(1-\frac{A(z^m)}{z}\right)\sum_{h=0}^{m-2}\frac{A(z^m)^h}{z^h}\prod_{l=1}^{h+1}B(z^l)-\frac{1}{z}\right] \tag{31}$$

Service of the tagged type-2 packet is delayed by the arrival of type-1 message (packet-train) with initial delay U. To derive the delayed busy period by type-1 messages, we assume that the service of type-1 messages is FCFS. Let θ be the delayed busy period generated by type-1 messages arriving during a slot. We can easily obtain the following equation in the same way that we derive the busy period of M/G/1 queue in the continuous-time case [8]:

$$\theta = X + \theta^{(1)} + \cdots + \theta^{(X)} \tag{32}$$

where X is the service time of messages arriving during a slot and $\theta^{(1)}$, $\theta^{(2)}$, ... are independent and have the same distribution as θ. Since a message consists of m packets, the service time of a message is m slots. Thus, the generating function $\theta(z)$ of the delayed busy period θ is given by $\theta(z) = A(z^m\theta^m(z))$. Therefore, the probability generating function $W_2(z)$ for the waiting time of a type-2 packet is given by

$$W_2(z) = U(z\theta(z)) \tag{33}$$

4.2 Waiting time distribution of type-2 message

Here we derive the waiting time distribution of a type-2 message (packet-train). We assume FCFS for messages in the same class. According to the service rule in Section 2, the type-1 message has pre-emptive priority over the type-2 message. Suppose a tagged type-2 message arrives in a slot S. Let $\tilde{a}_k$ be the number of the kth packet arrivals among the type-1 packet-trains currently arriving in the slot S. Obviously, $\tilde{a}_k$ has the same distribution as a_k, $1 \le k \le m$. The unfinished work $\tilde{U}$ at the beginning of the slot $S + 1$ for the tagged type-2 message can be expressed by the system state in the slot S:

$$\tilde{U} = (N_1 + N_2 - 1)^+ + m\tilde{a}_1 + (m-1)\tilde{a}_2 + \cdots + \tilde{a}_m + (m-1)b_1 + \cdots + b_{m-1} + mf \tag{34}$$

where f is the service order of the tagged type-2 messages among the type-2 messages arriving in the slot S with the probability generating function $F(z)$. The probability generating function $\tilde{U}(z)$ of the unfinished work $\tilde{U}$ can then be calculated as follows:

$$\tilde{U}(z)$$

$$= E\Big[z^{(N_1+N_2-1)^+ + m\tilde{a}_1+\cdots+\tilde{a}_m} \times z^{(m-1)b_1+\cdots+b_{m-1}+mf}\Big]$$

$$= E\Big[z^{(N_1+N_2-1)^+ + m\tilde{a}_1+\cdots+\tilde{a}_m} \times z^{(m-1)b_1+\cdots+b_{m-1}+mf}1_{\{N_1\neq 0 \text{ or } N_2\neq 0\}}\Big]$$

$$+ E\Big[z^{(N_1+N_2-1)^+ + m\tilde{a}_1+\cdots+\tilde{a}_m} \times z^{(m-1)b_1+\cdots+b_{m-1}+mf}1_{\{N_1=0,\ N_2=0\}}\Big]$$

$$= \frac{A(z^m)F(z^m)}{z}\Big[zp_0 + E\Big[z^{(m-1)a_1+\cdots+a_{m-1}+N_1} \times z^{(m-1)b_1+\cdots+b_{m-1}+N_2}1_{\{N_1\neq 0 \text{ or } N_2\neq 0\}}\Big]\Big]$$

$$= \frac{1}{z}A(z^m)F(z^m)R(z^{m-1},\ldots,z,z,z^{m-1},\ldots,z,z)$$

$$+ A(z^m)F(z^m)\left(1-\frac{1}{z}\right)p_0 \tag{35}$$

By setting $x_k = y_k = z^{m-k}$, $k = 1, \ldots, m-1$, in eqn. 10, we obtain

$$R(z^{m-1},\ldots,z,z^{m-1},\ldots,z) = \frac{A(z^m)B(z^m)(z-1)p_0}{z-A(z^m)B(z^m)} \tag{36}$$

By substituting the above result into eqn. 32, we can finally derive

$$\tilde{U}(z) = A(z^m)F(z^m)\left[\left(1-\frac{1}{z}\right)\frac{A(z^m)B(z^m)}{z-A(z^m)B(z^m)} + \left(1-\frac{1}{z}\right)\right]p_0$$

$$= A(z^m)F(z^m)\left(1-\frac{1}{z}\right)\frac{z}{z-A(z^m)B(z^m)}p_0$$

$$= \frac{A(z^m)F(z^m)(z-1)}{z-A(z^m)B(z^m)}p_0 \tag{37}$$

On the other hand, the probability generating function $F(z)$ of the random variable f [9] is given easily by

$$F(z) = \frac{B(z)-1}{(z-1)B'(1)} \tag{38}$$

Therefore, we obtain the probability generating function $W_2^M(z)$ for the waiting time of a type-2 message:

$$W_2^M(z) = \tilde{U}(z\theta(z)) \quad (39)$$

where $\theta(z)$ is the delayed busy period generated by type-1 messages arriving during a slot, as in Section 4.1.

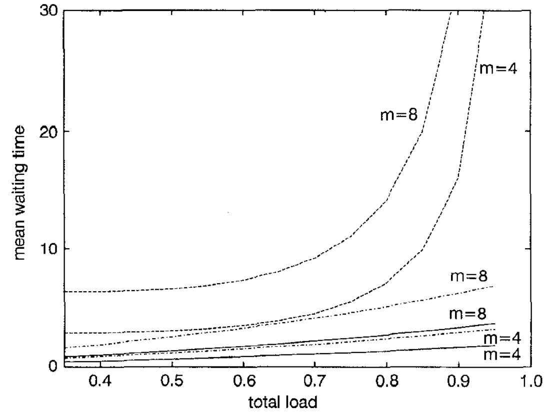

Fig. 1 *Mean waiting time against total load for $\rho_1 = \rho_2$ and m = 4, 8*
——— type-1
- - - - type 2
- · - · - no priority

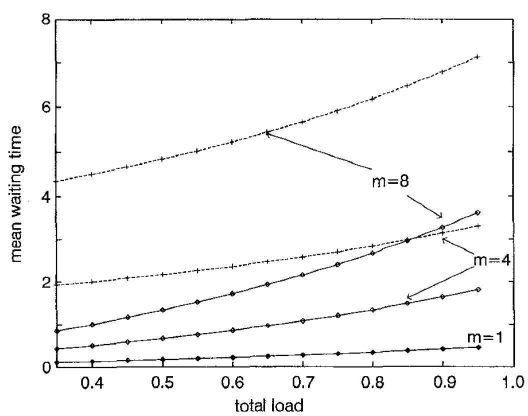

Fig. 2 *Mean waiting time of type-1 against total load for $\rho_1 = \rho_2$ and m = 1, 4, 8*
◇ train
+ batch

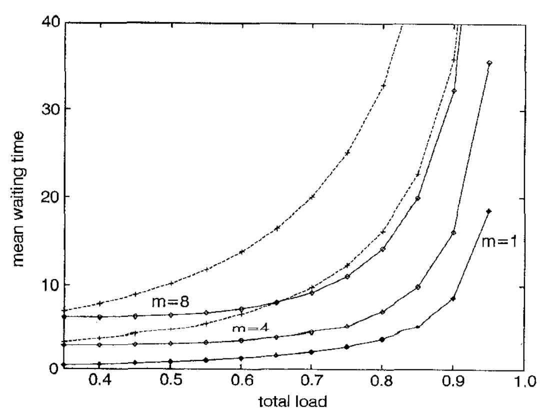

Fig. 3 *Mean waiting time of type-2 against total load for $\rho_1 = \rho_2$ and m = 1, 4, 8*
◇ train
+ batch

5 Numerical examples

We consider a statistical multiplexer, at which fixed-length packet-trains arrive. Assume that the number of arriving type-i messages per slot follows a Poisson distribution with parameter p_i, so that the probability generating functions are given by

$$A(z) = e^{p_1(z-1)}$$
$$B(z) = e^{p_2(z-1)} \quad (40)$$

In Fig. 1, $E[W_1]$ and $E[W_2]$ are shown as a function of ρ for $\rho_1 = \rho_2$ and m = 4, 8. The mean waiting times for a queueing system with no priority are displayed together for comparision. Note that the mean waiting time of type-1 packets is reduced at the expense of an increased mean waiting time of type-2 packets. As expected intuitively, the mean waiting time is an increasing function of m for a given value of ρ, i.e. for a given total load, fewer long messages result in longer waiting times.

We also compare the train arrivals, where messages enter the buffer at the rate of one packet per slot, with the batch arrivals, where all the packets of a message enter the buffer during the same slot. The waiting time in the case of batch arrivals can be calculated [4]. In Figs. 2 and 3, $E[W_1]$ and $E[W_2]$ are plotted against ρ for Poisson train and Poisson batch arrival processes, $\rho_1 = \rho_2$ and m = 1, 4, 8. Figs. 2 and 3 show that the mean waiting times for the batch arrivals are much greater than those for the train arrivals. Thus, we conclude that the batch arrival assumption for messages is not reasonable and, in practice, the impact of the train arrival on performance cannot be ignored.

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